Geomechanics

LECTURE 13

Time Dependent Behaviour of Geomaterials

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Laboratory of soil mechanics – Fall 2025

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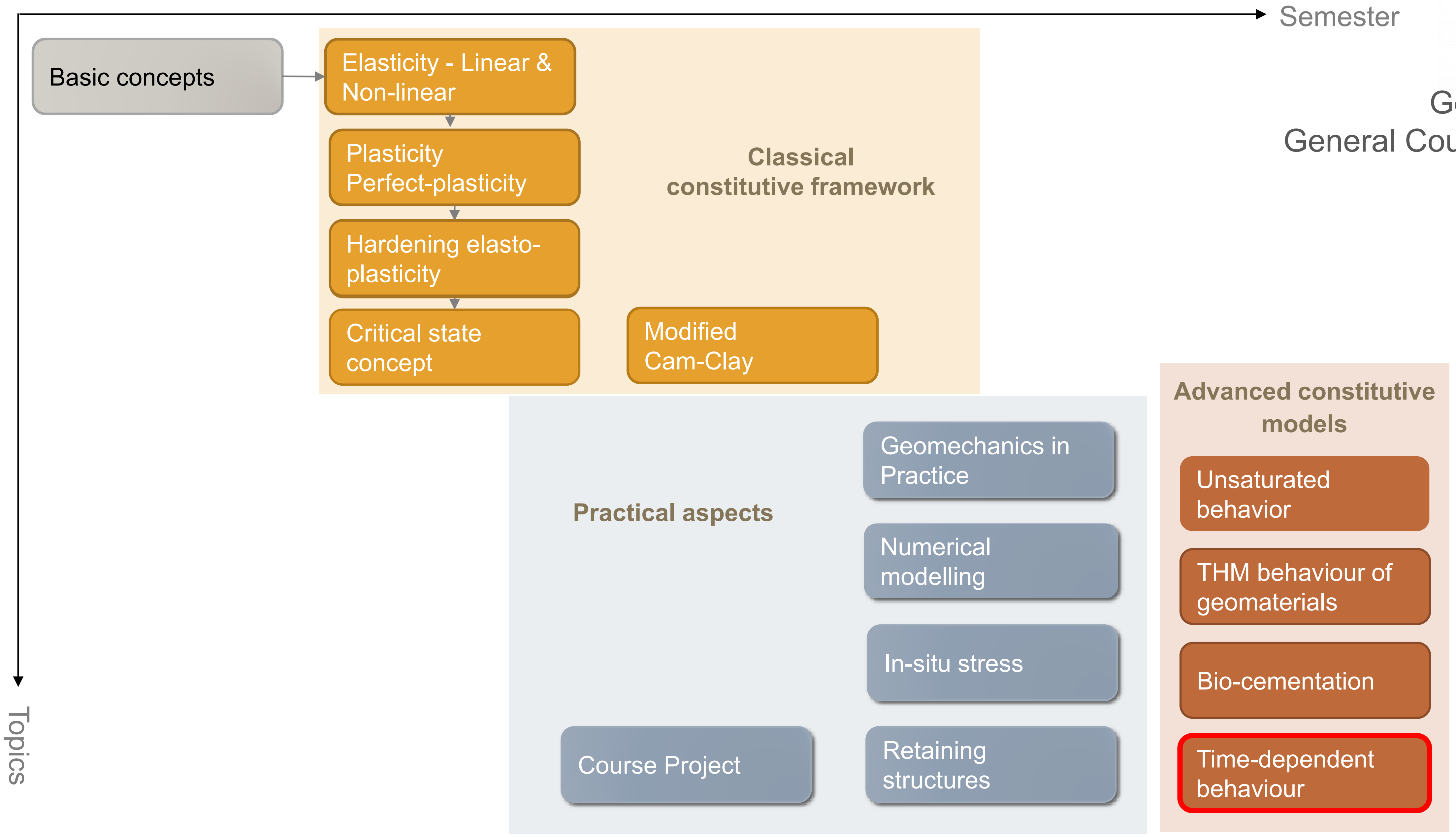
Geomechanics
General Course overview



Content

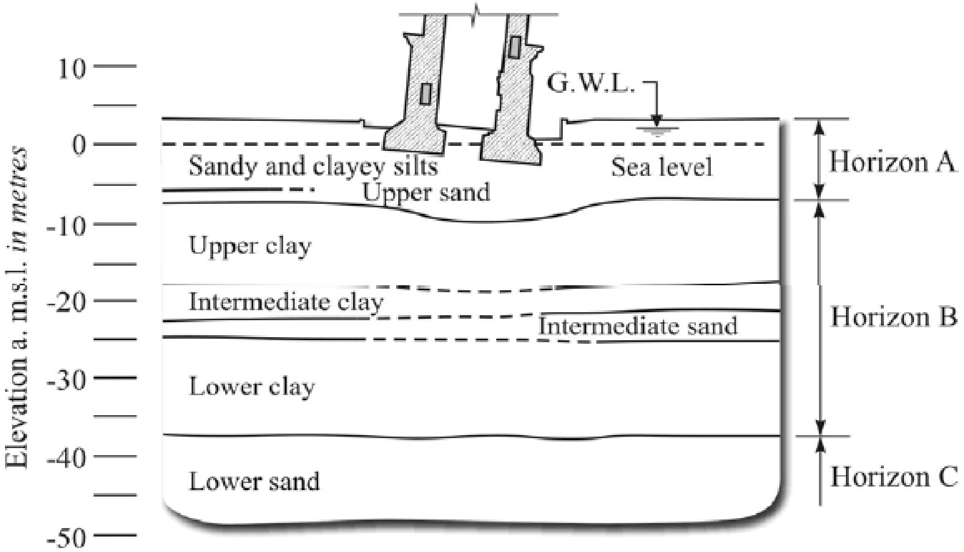
- Example of real cases
- Hydro-mechanical vs viscous response
- Rheological aspects
- Experimental evidence of viscous behaviour
- Constitutive modelling for visco-elasto-plasticity

Real cases

Leaning Tower of Pisa

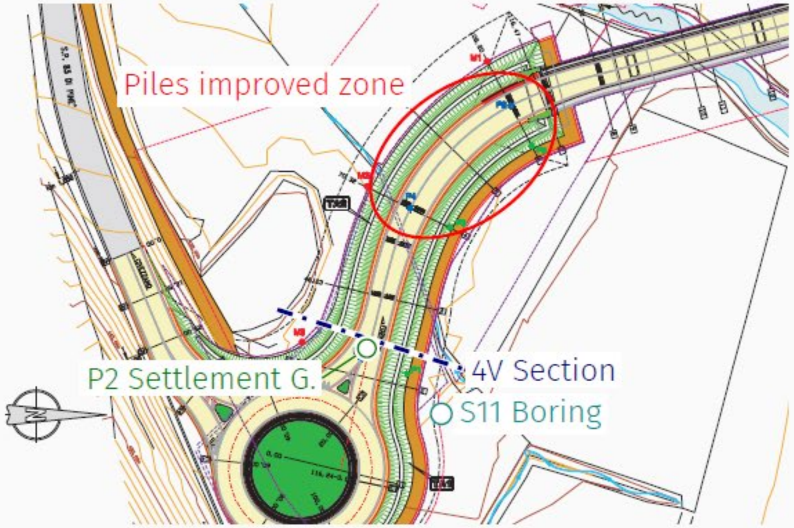
Causes of the leaning

- Creep (200 years for completing the construction)
- Non-homogeneous soil, mostly normally consolidated or slightly over consolidated clays
- Structural issues



Embankment foundation in Trento

Field evidence: road embankment founded on a **thick layer of organic clay** (Trento, Italy).



Madaschi and Gajo (2015)

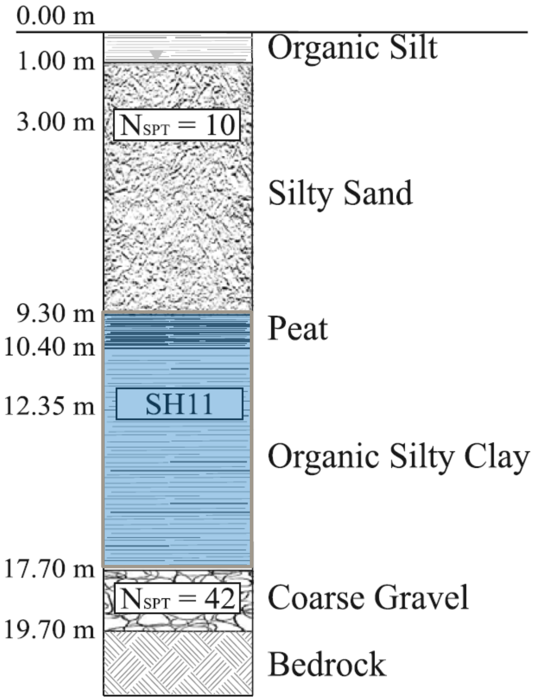
Problem description:

- Settlement gauges installed at the beginning of the construction measured **1 m of settlement after less than 1 year**
- At bridge abutment, the embankment foundation is reinforced with piles

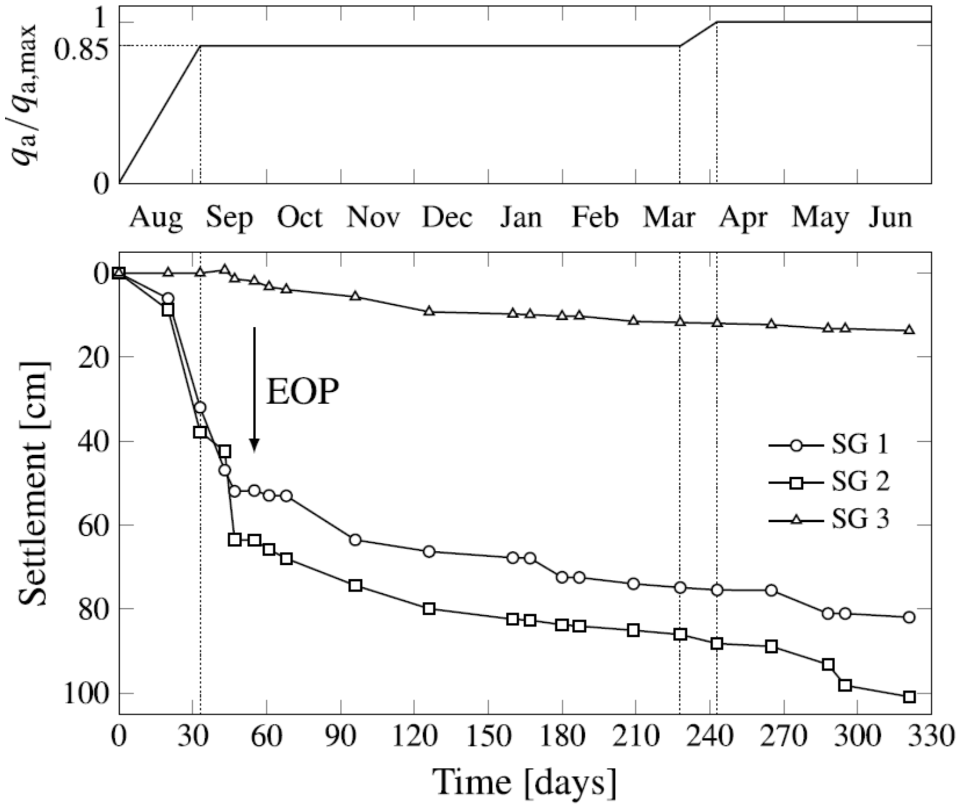
Embankment foundation in Trento

The **organic layer is 9 m thick** and the settlement at section 4V is **more than 1 m after 1 year.**

Stratigraphy – Boring S11



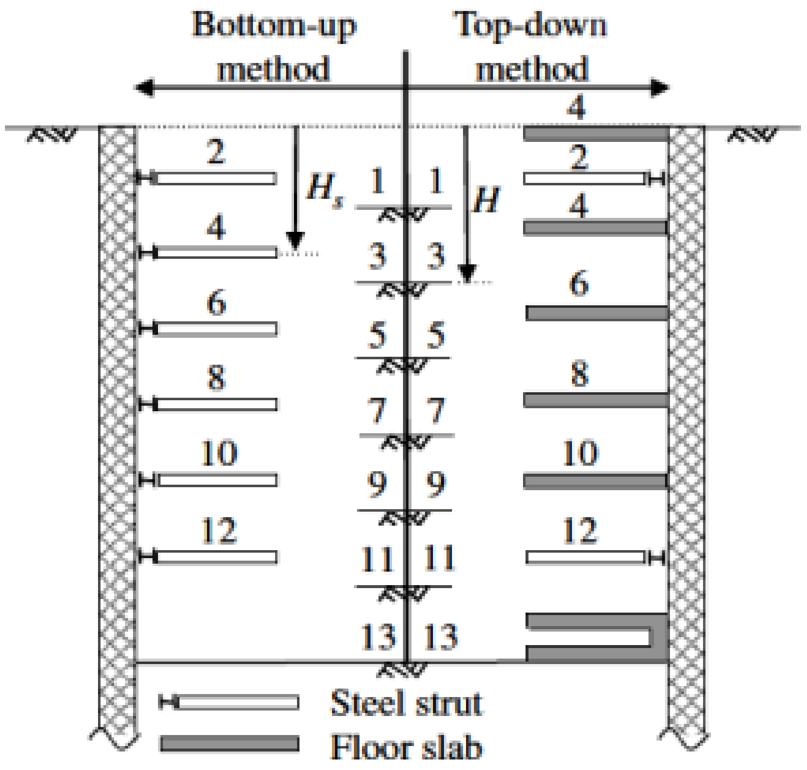
Load and settlement history



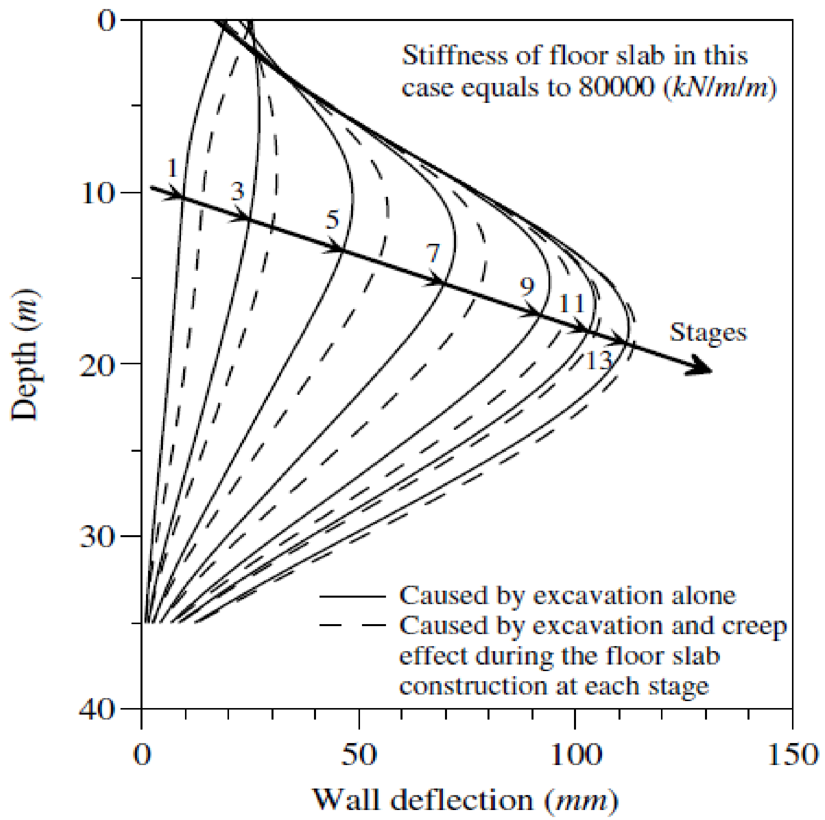
EOP: end of the primary consolidation

Madaschi and Gajo (2015)

Diaphragm wall in Taipei



Note: Numbers represent excavation stages



Deflection increases due to the **longer construction duration** in top-down method (Kung, 2009).

Mexico city

Extensive pumping to extract water from Mexico City's subsoil has caused regional sinking

Pumping

↓

Water table decreases

↓

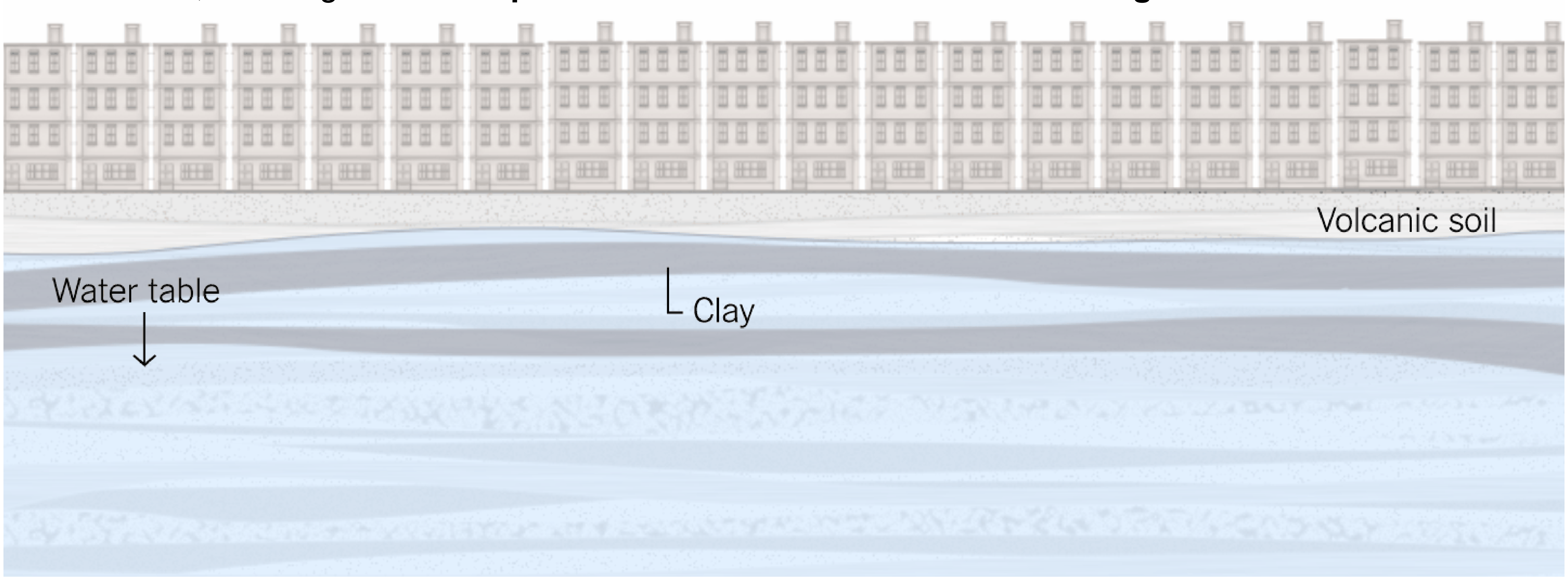
Effective stress increases

↓

Soil compaction

Mexico city

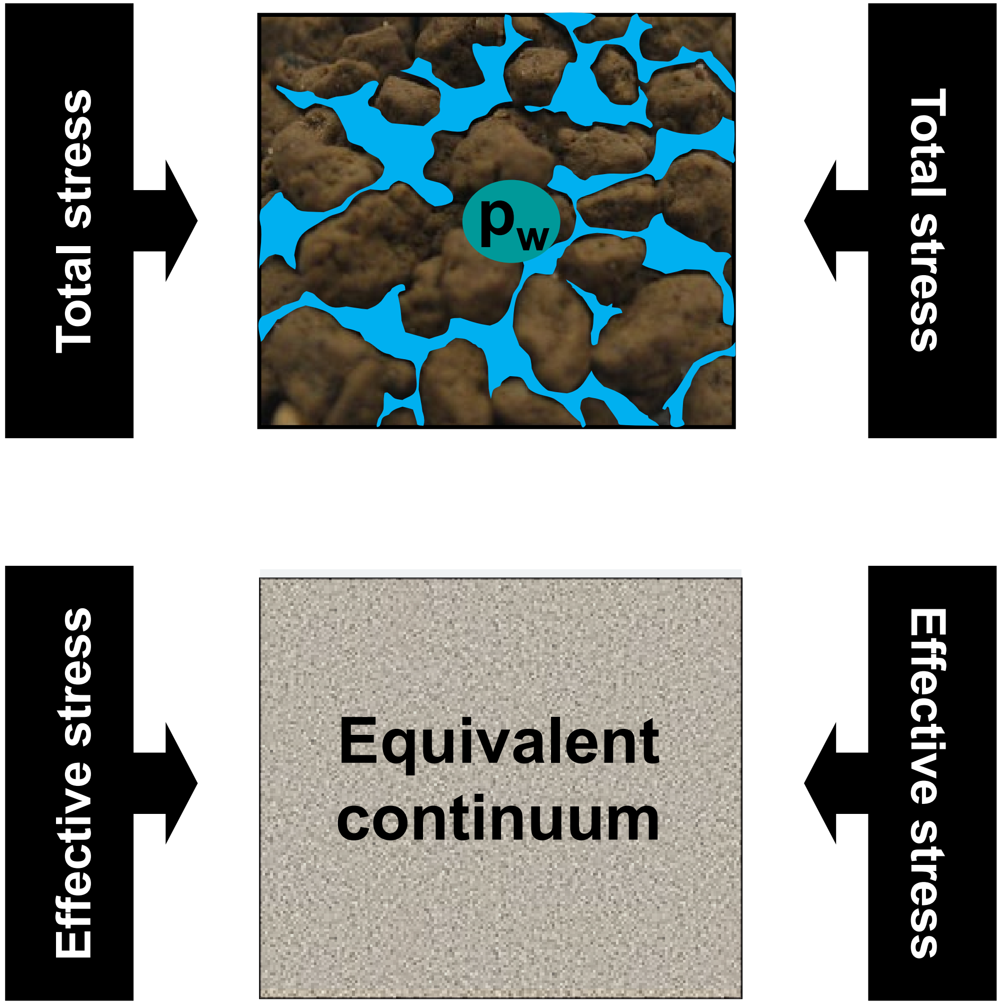
The city **pumps water** from the underground, the **water table decreases** and so the **effective stress increases**, inducing a **soil compaction** – uneven because of the **inhomogeneous soil strata**.



The New York Times
https://www.nytimes.com/interactive/2017/02/17/world/americas/mexico-city-sinking.html

Hydro-mechanical vs viscous response

Hydro-mechanical response: effective stress



- **Terzaghi's effective stress** (1936)

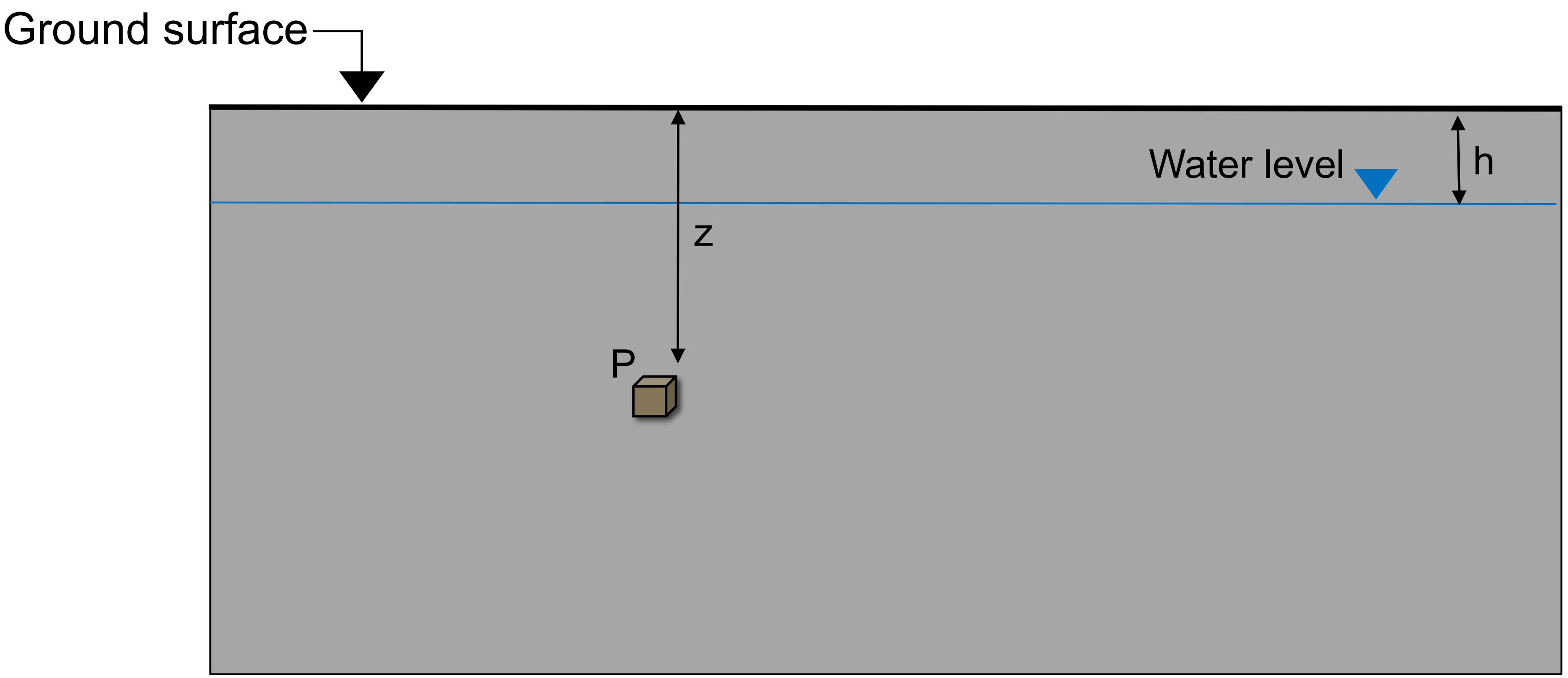
$$\sigma'_{ij} = \sigma_{ij} - p_w \delta_{ij}$$

Total stress

Pore water pressure

- **Assumptions**
  - Fully saturated granular material
  - Incompressible fluid and grains
- **All measurable effects produced by a change in the state of stress are due to a change in the effective stress** (Terzaghi, 1936)

Hydro-mechanical response: effective stress

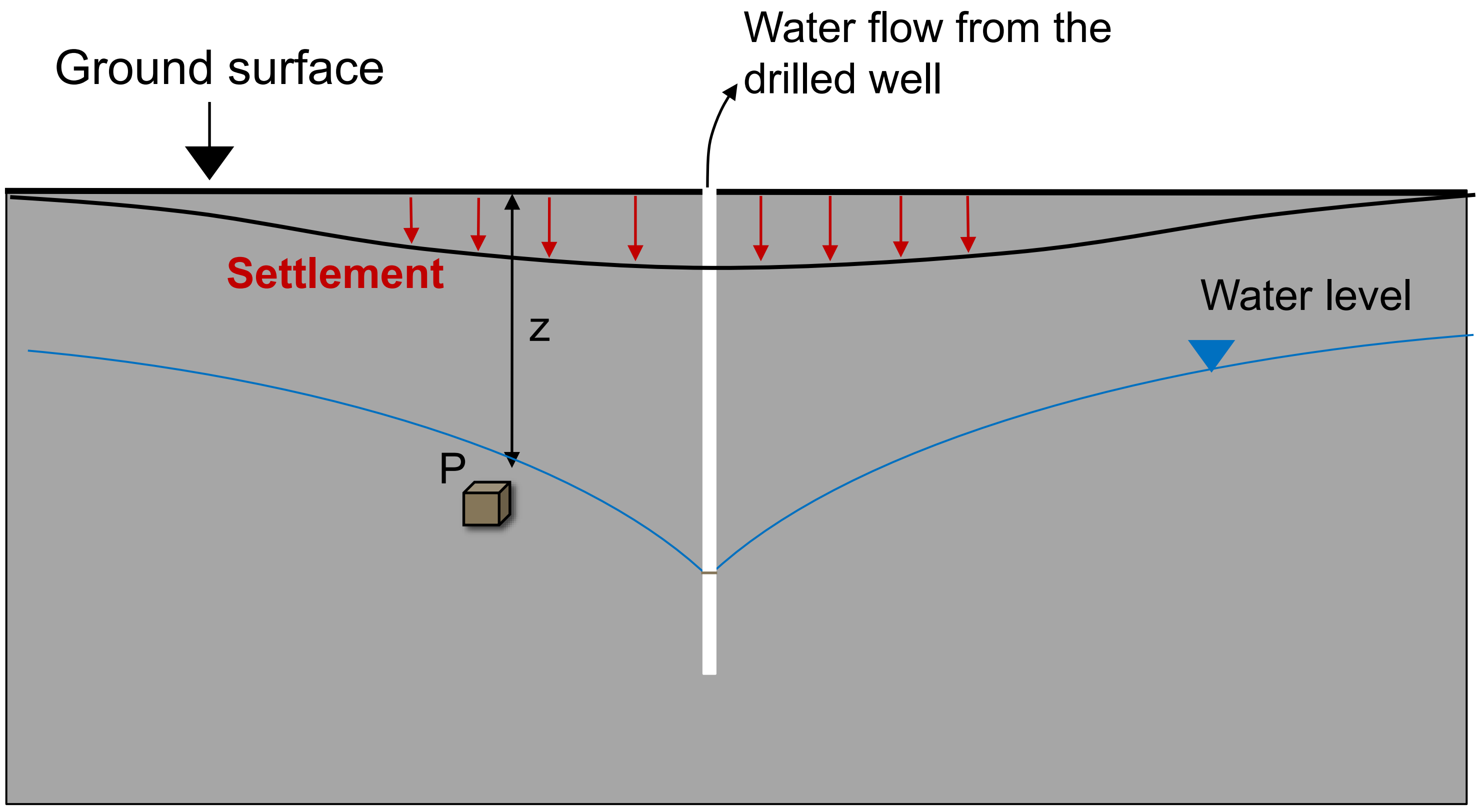


Stress state at point P:

$$\left.\begin{aligned} \sigma_v &= \gamma_{sat} z \\ p_w &= \gamma_w (z - h) \end{aligned}\right\} \quad \sigma_v' = \sigma_v - p_w$$

$$\sigma_h' = K_0 \sigma'_v$$

Hydro-mechanical response: effective stress



Pore water pressure decrease at point P causes an increase of effective stress

$$d\sigma'_{ij} = d\sigma_{ij} - dp_w\delta_{ij}$$

Deformation (i.e. settlements) are induced

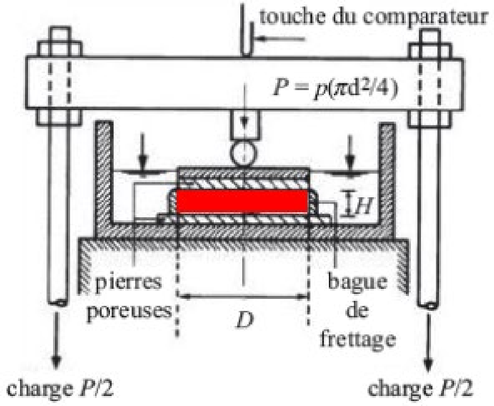
$$d\varepsilon_{kl} = C_{ijkl}d\sigma'_{ij}$$

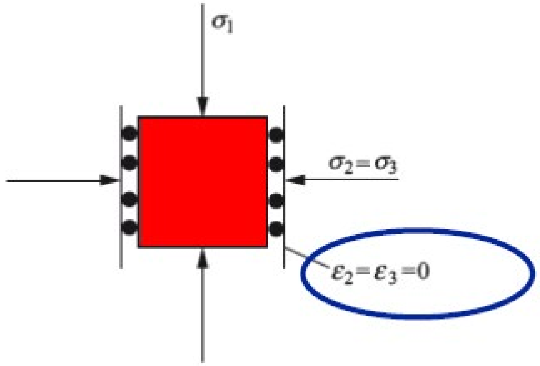
Hydro-mechanical response

Oedometric test

Usually performed to analyze the vertical settlement problem related to H-M coupling

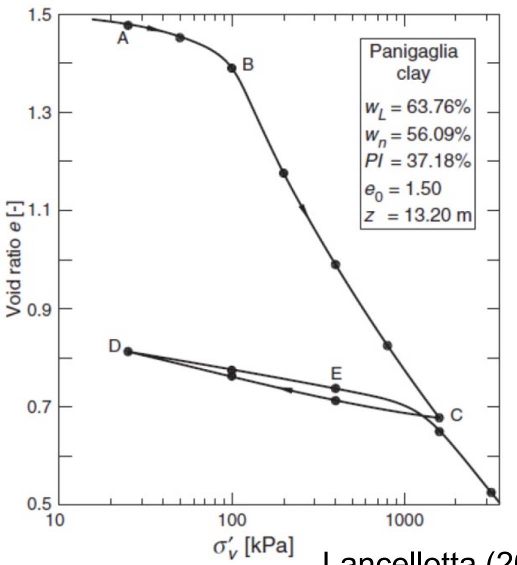
- 1D consolidation
- Load applied in steps
- Analysis of the settlement versus time





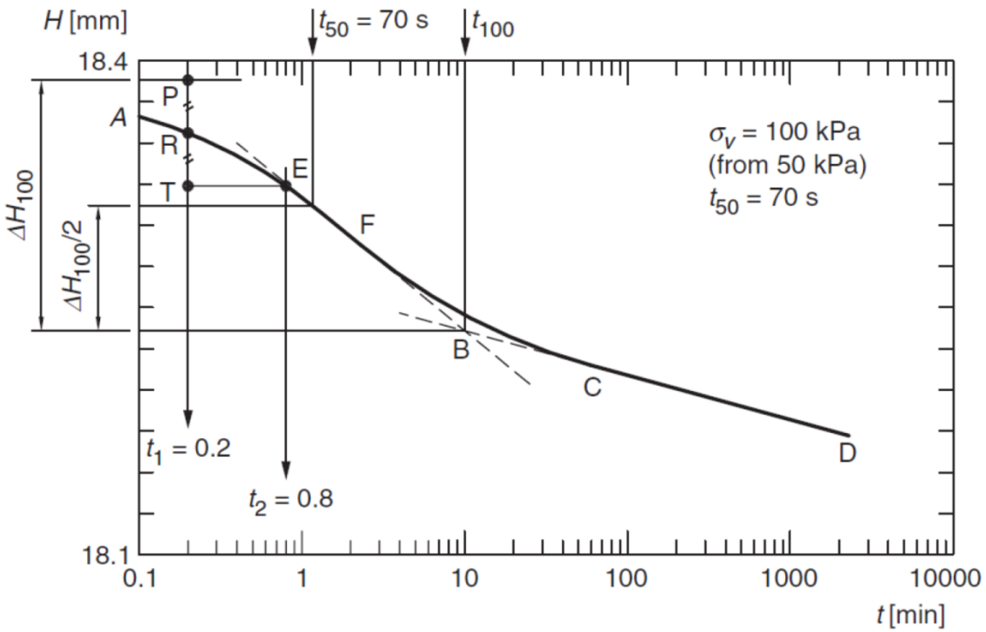
TGC 18

e – log σ_v



Lancellotta (2009)

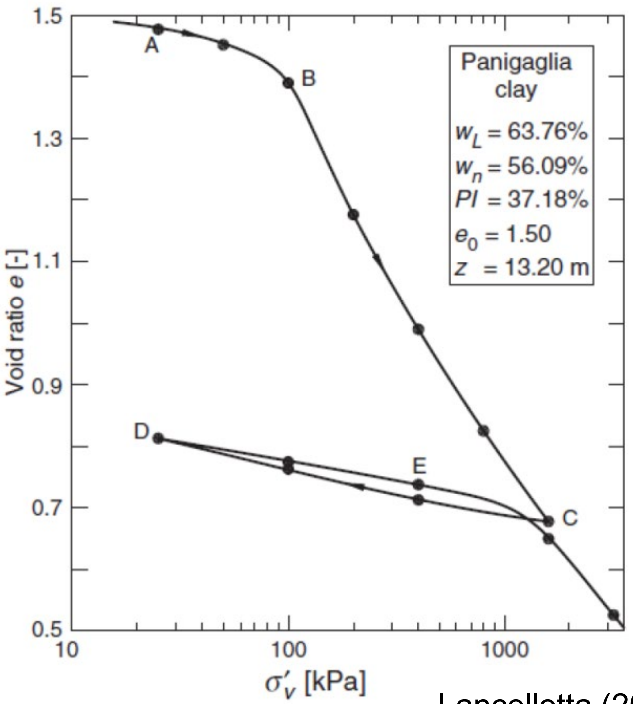
H – time



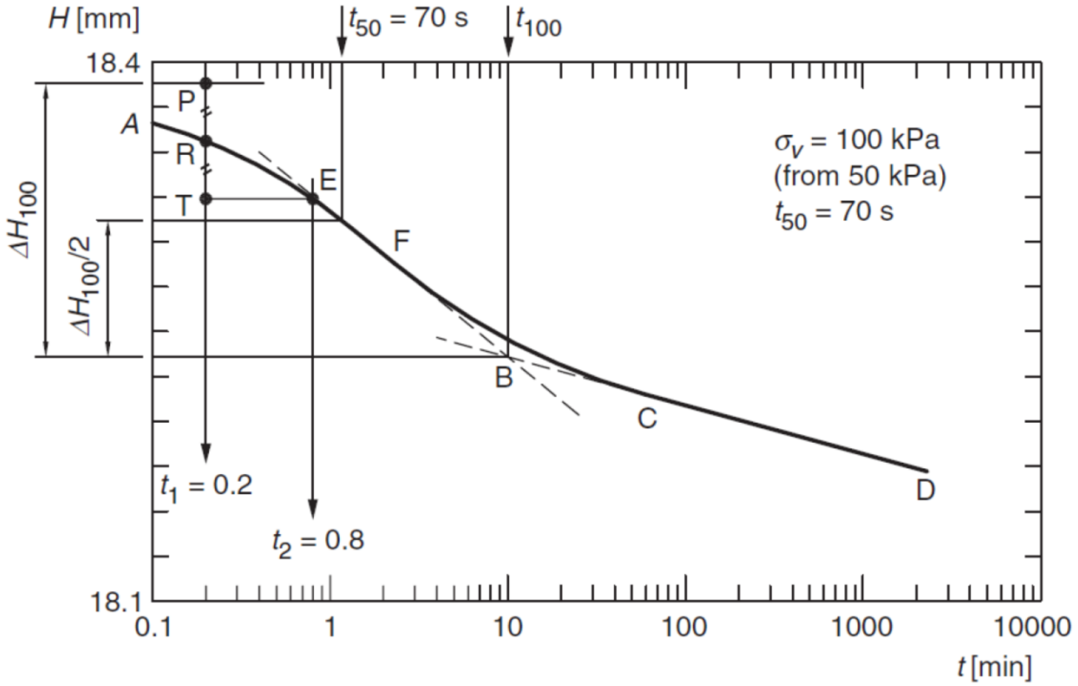
Hydro-mechanical response

Oedometric test: settlement curve

- **Instantaneous** application of the load
- **Initial undrained response** of the material (generation of the excess pore water pressure)
- **Dissipation** in time of the excess of pore water pressure (drained process)
- **Consolidation** of the material during time → settlement



Lancellotta (2009)



Lancellotta (2009)

Hydro-mechanical response

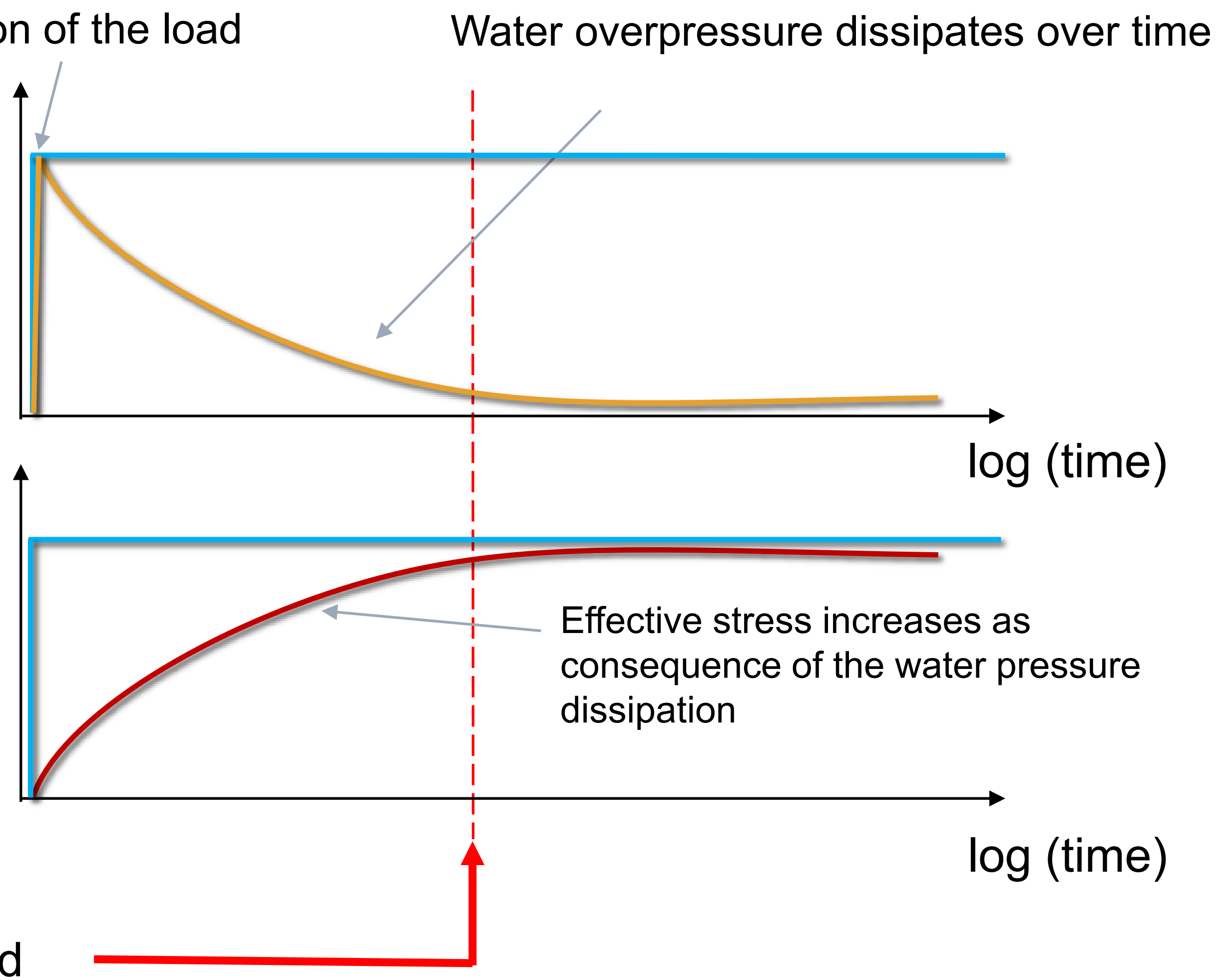
Oedometric test: settlement curve

$\Delta\sigma$ – total stress

Δp_w – pore water overpressure

$\Delta\sigma$ – total stress

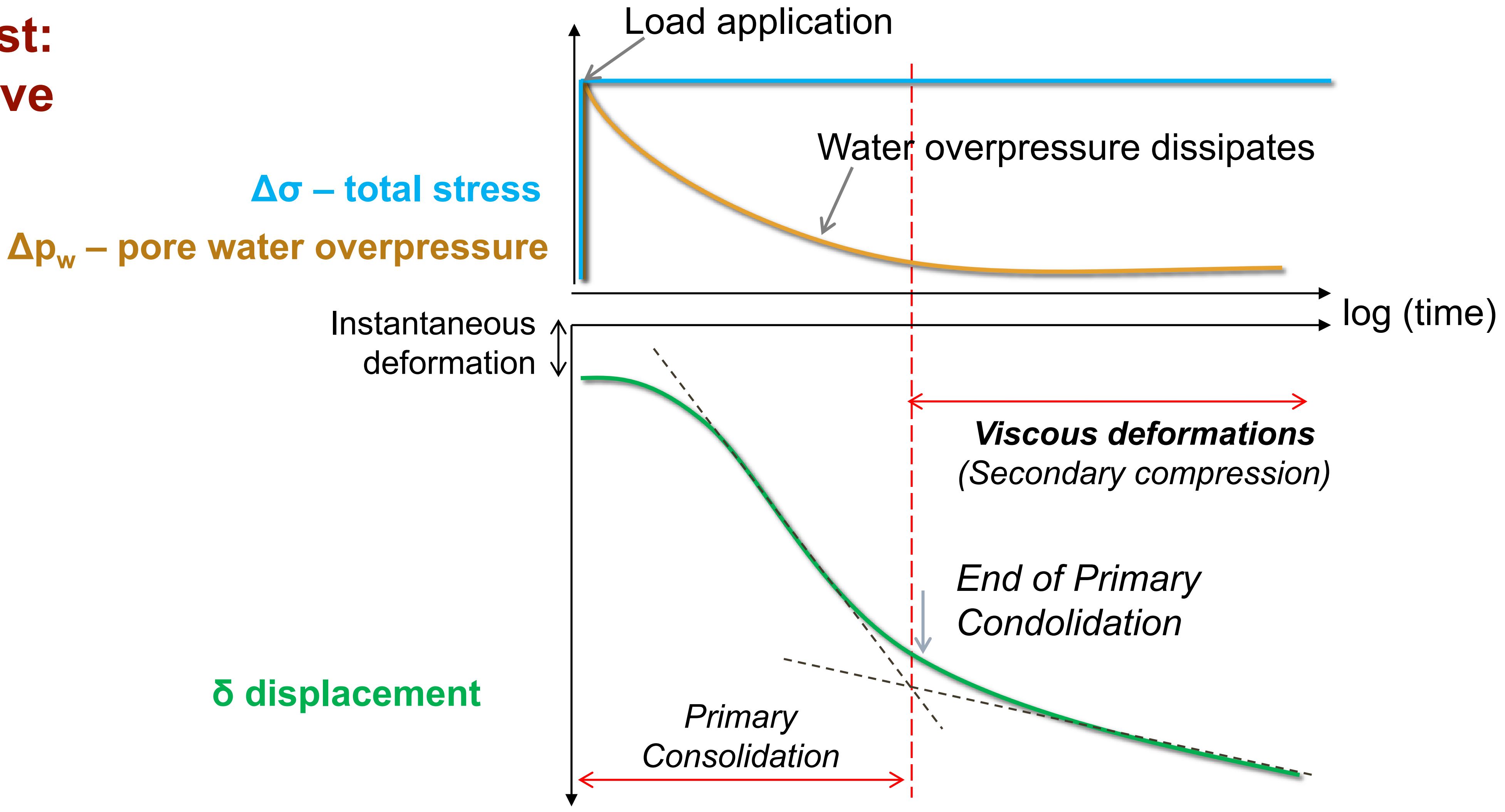
$\Delta\sigma'$ – effective stress



End of the consolidation process:
- pore water pressure is completely dissipated
- the material is subjected to a constant stress

Hydro-mechanical response

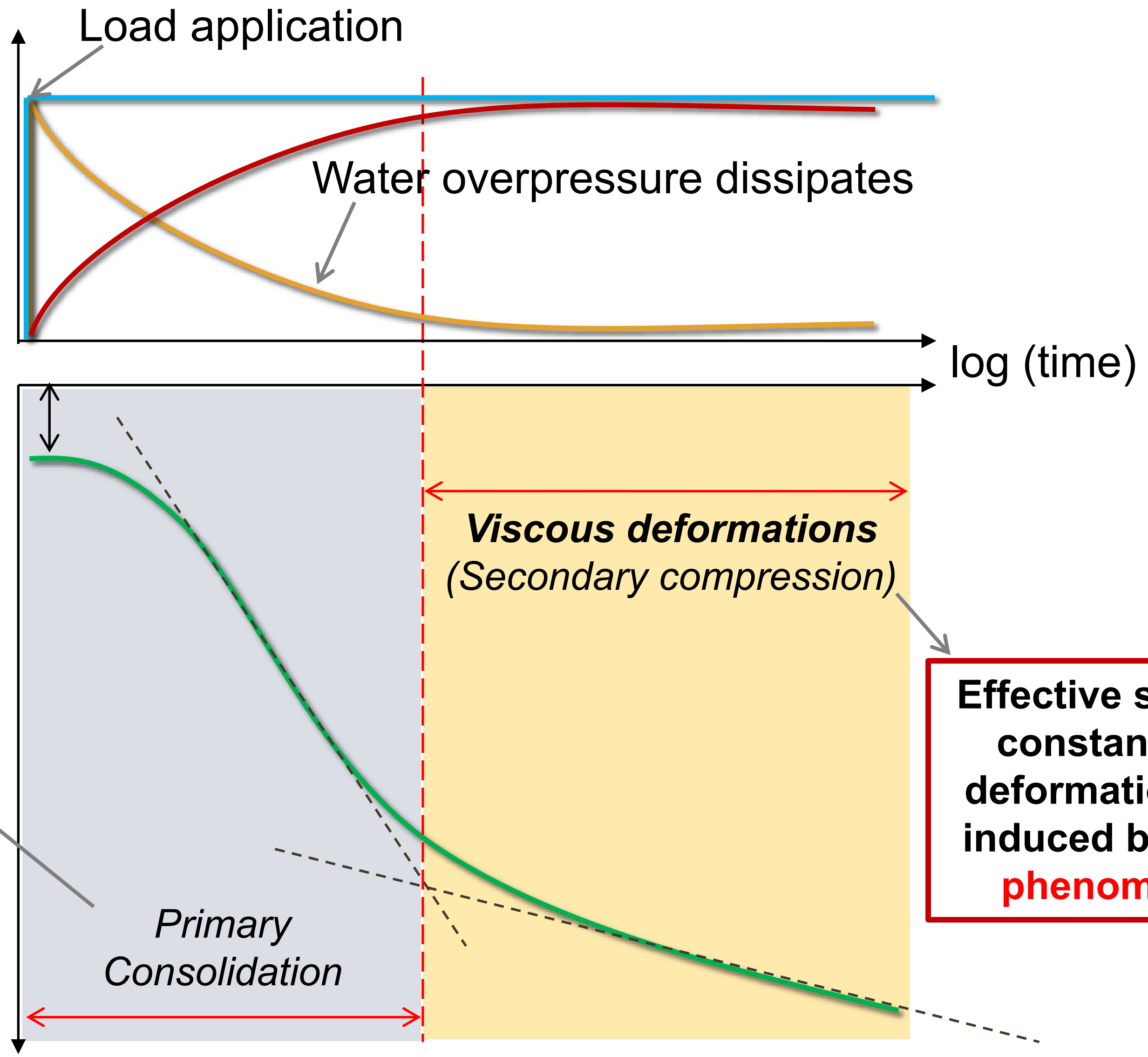
Oedometric test: settlement curve



Hydro-mechanical response

Oedometric test: settlement curve

Δσ – total stress
Δp_w – pore water overpressure
Δσ' – effective stress

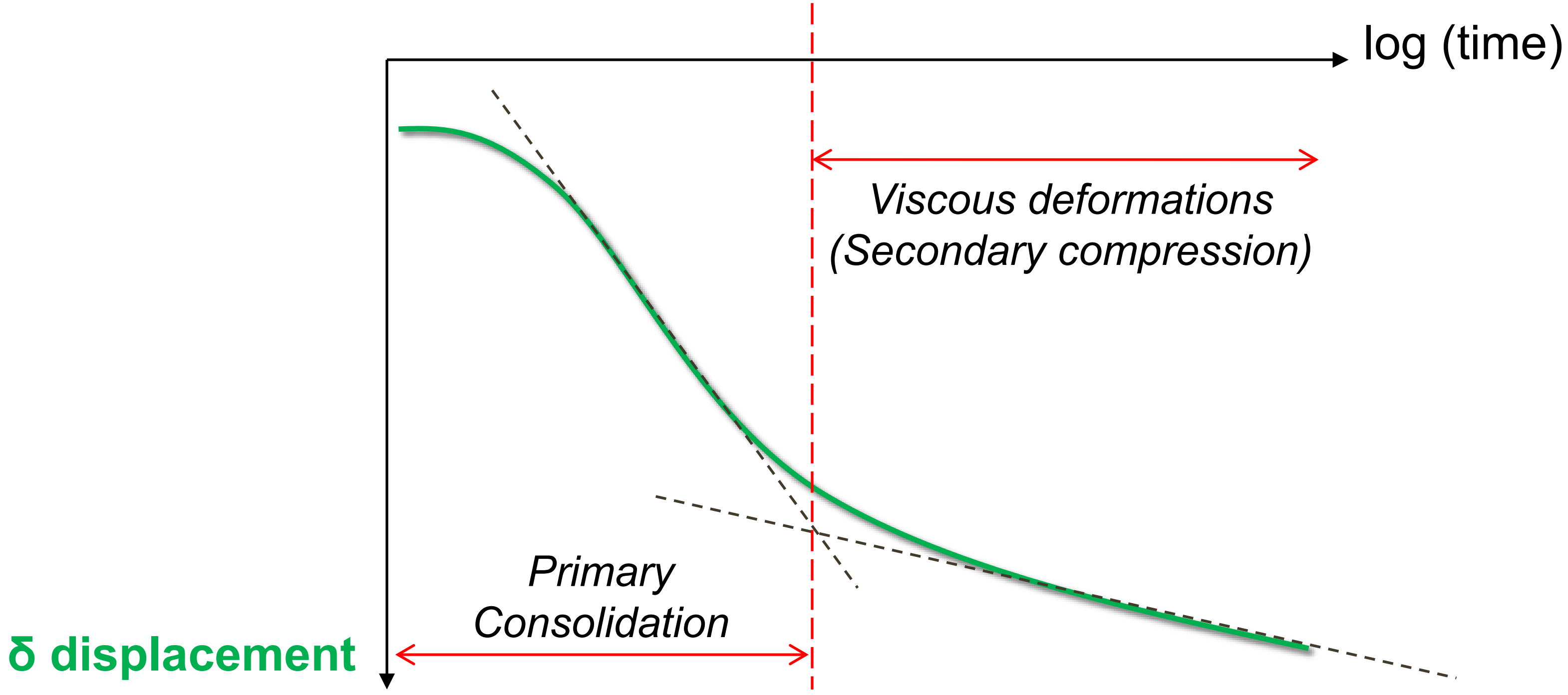


The deformations are induced by the change in effective stress

Effective stress is constant. The deformations are induced by creep phenomenon

δ displacement

Viscous deformations



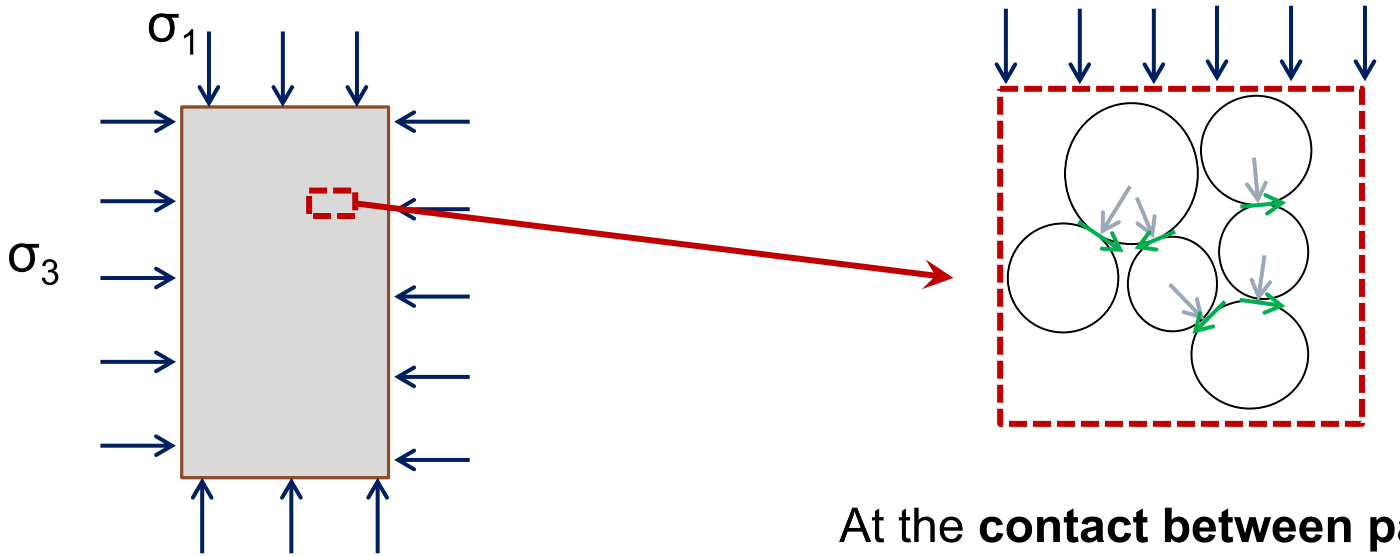
Viscous deformations
=
Time dependent deformations

Rheological aspects

Viscous behaviour

Macroscopic stress distribution in a laboratory sample

Microscopic structure of a sample

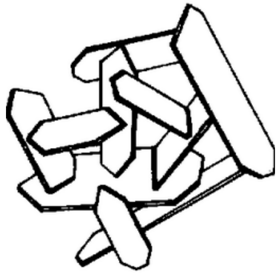


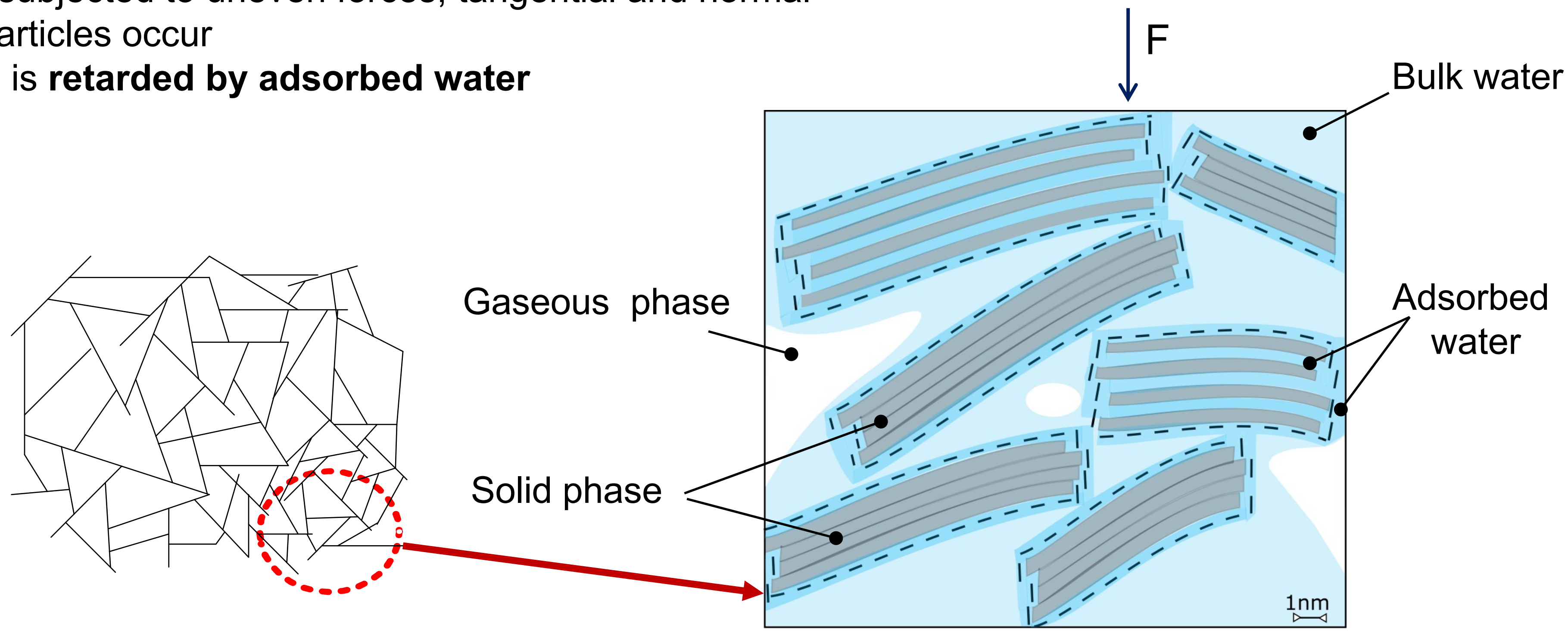
At the **contact between particles**, shear and normal forces are generated

Viscous behaviour

Fine grained geomaterials

- Structure of clay is **card-house structure**
- The **macroscopic stress is microscopically distributed** in force chains
- The **particle contact** are subjected to uneven forces, tangential and normal
- **Sliding and rotation** of particles occur
- Contact between particles is **retarded by adsorbed water**





Experimental evidence of the viscous behaviour

Rate and Time Dependency

Time dependent effects affect the response of many engineering applications

In general, **three conditions** are identified in the field of viscous phenomena:

- **Creep**: deformation under constant load conditions
- **Relaxation**: stress decrement under constant strain conditions
- **Strain rate**: response dependent on the applied strain rate
  (change in deformation with respect to time)

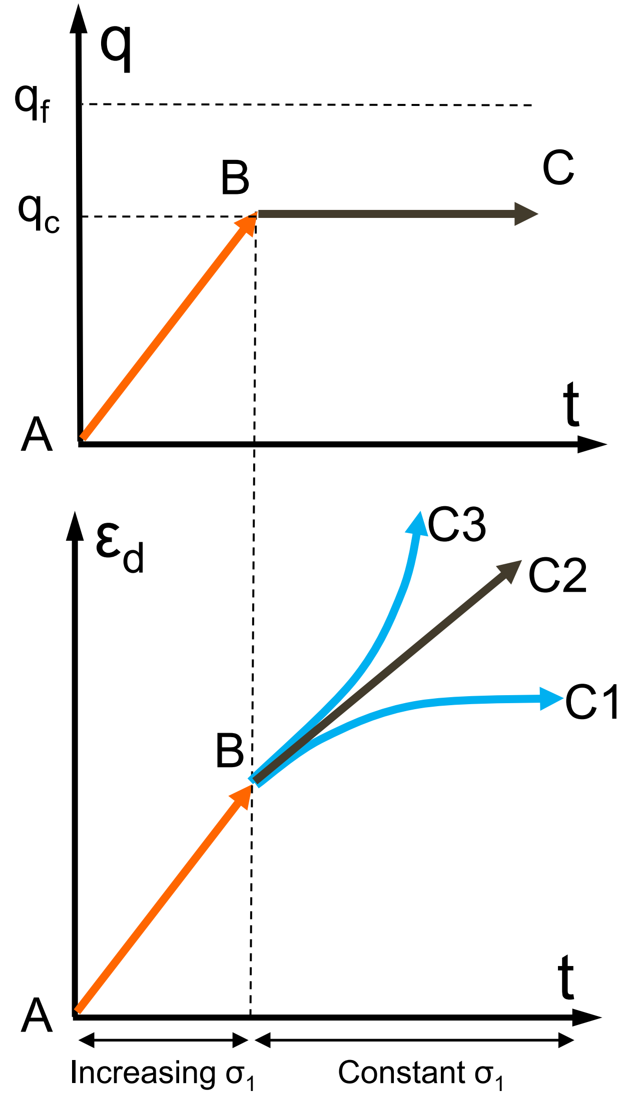
Both **triaxial** and **oedometric** tests can be used to investigate the viscous behaviour of geomaterials

Triaxial testing - Creep

CREEP: Time dependency of strains

Triaxial tests terminated at a **deviatoric stress q_c** lower than the deviatoric stress corresponding to failure conditions q_f and then **maintained constant**

Response at constant deviatoric stress q_c



Deviatoric strain increases in time at:

- Decaying rate (C1)
- Constant rate (C2)
- Accelerating rate (C3)

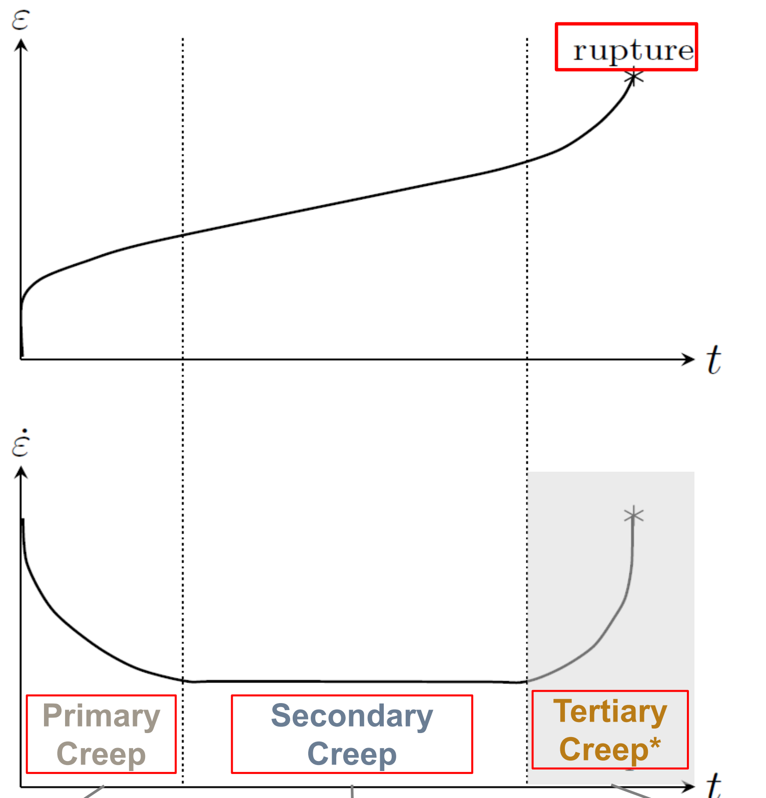
Depending on the ratio $[q_c / q_f]$

Triaxial testing - Creep

Triaxial Creep test

Deformation
under constant load

Strain rate
Under constant load



The strain rate decreases

The strain rate is constant

The strain rate increases

Augusteesen et al. (2014)

Depending on the analysed material it is possible to identify some **common features of the time dependent response**

The typical time dependent deformation of engineering materials under constant load shows **three characteristic phases.**

* The presence of the tertiary phase depends on the material and on the stress level

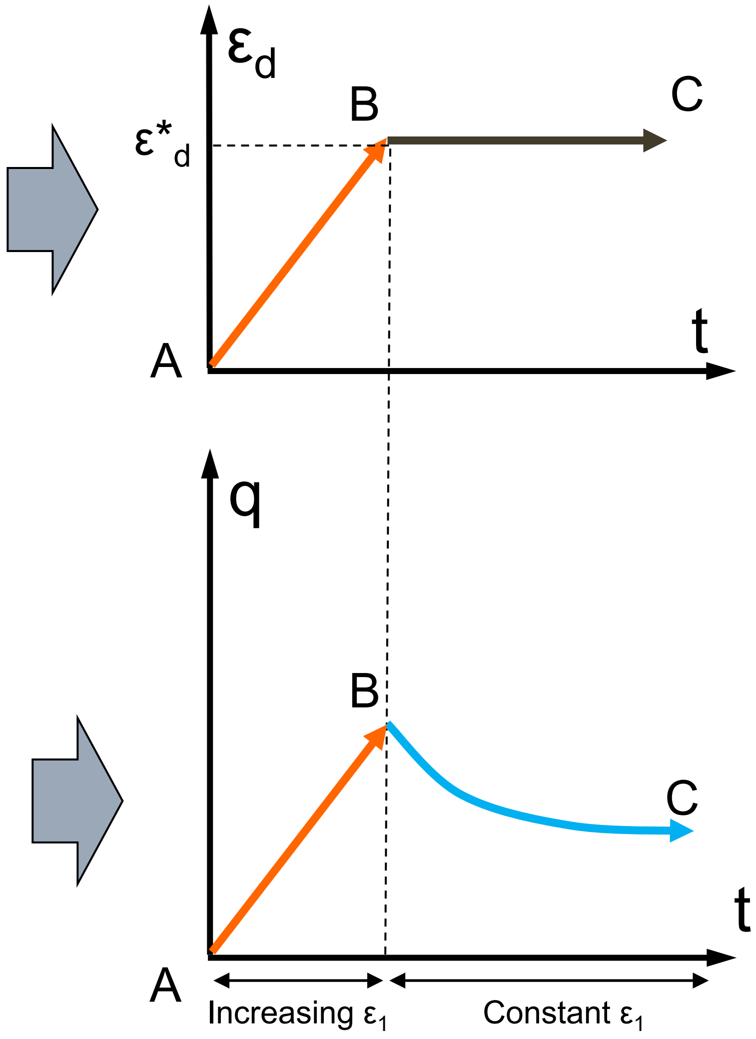
Triaxial testing - relaxation

RELAXATION: Time dependency of stresses

Triaxial tests terminated at a given **deviatoric strain ε^*_d** and then maintained constant

Response at constant deviatoric stress ε^*_d

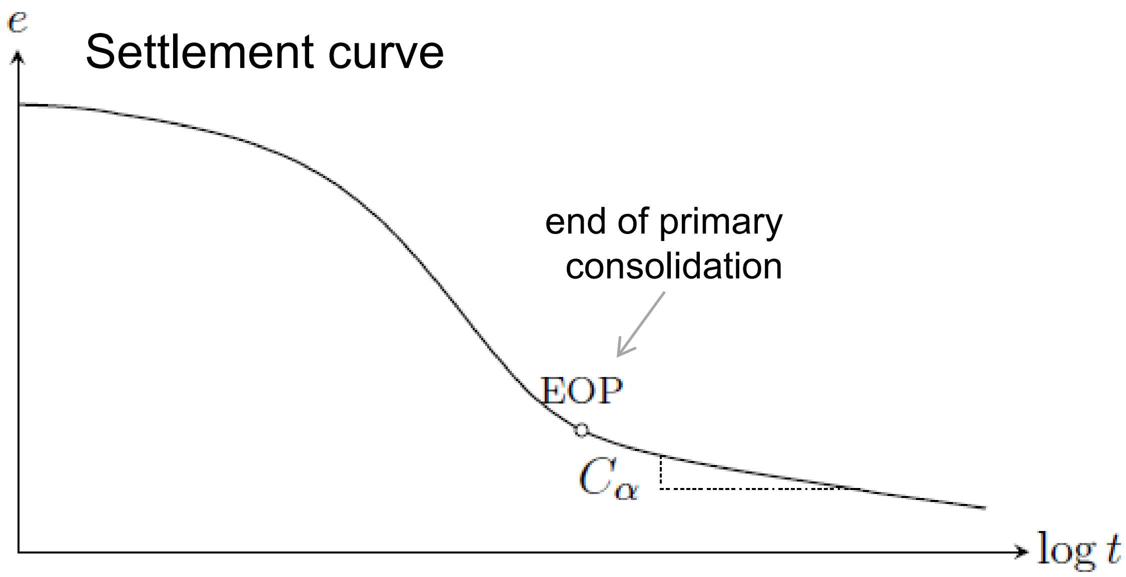
Deviatoric stress [q] decreases at constant deviatoric strain [ε^*_d]



Oedometric testing

The most common experimental test to assess the compression behaviour of soils is the **oedometric test**.

One of the simplest parameters to describe viscous behaviour of geomaterials is the **coefficient of secondary compression (C_α)**, defined as the slope of the oedometer curve in $e - \log t$ plot at the end of the primary consolidation



Viscous parameters

$$C_\alpha = -\frac{\Delta e}{\Delta \log t} \qquad C_{\alpha\varepsilon} = \frac{\Delta \varepsilon}{\Delta \log t} = \frac{C_\alpha}{1 + e_0}$$

From C_α it is possible to estimate the **amount of viscous deformation** of the soil:

$$\varepsilon_z^v(t) = C_{\alpha\varepsilon} \log\left(1 + \frac{t}{t_i}\right)$$

Oedometric testing

Typical C_α/C_c values for geomaterials

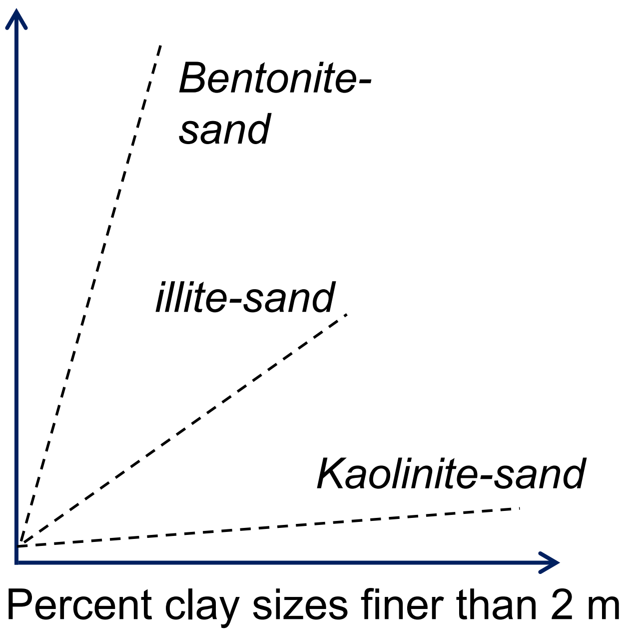
C_α/C_c **ratio** is usually used to characterize the importance of the viscous behaviour compared to the consolidation process

Table 16.1 Values of C_α/C_c for Geotechnical Materials

Material	C_α/C_c
Granular soils including rockfill	0.02 ± 0.01
Shale and mudstone	0.03 ± 0.01
Inorganic clays and silts	0.04 ± 0.01
Organic clays and silts	0.05 ± 0.01
Peat and muskeg	0.06 ± 0.01

Terzaghi, Peck and Mesri (1996)

Creep rate strongly depend on the nature of clay. **The smaller the particle, the greater the specific surface area** (i.e. surface area per unit mass of solid).



Steady state creep rate

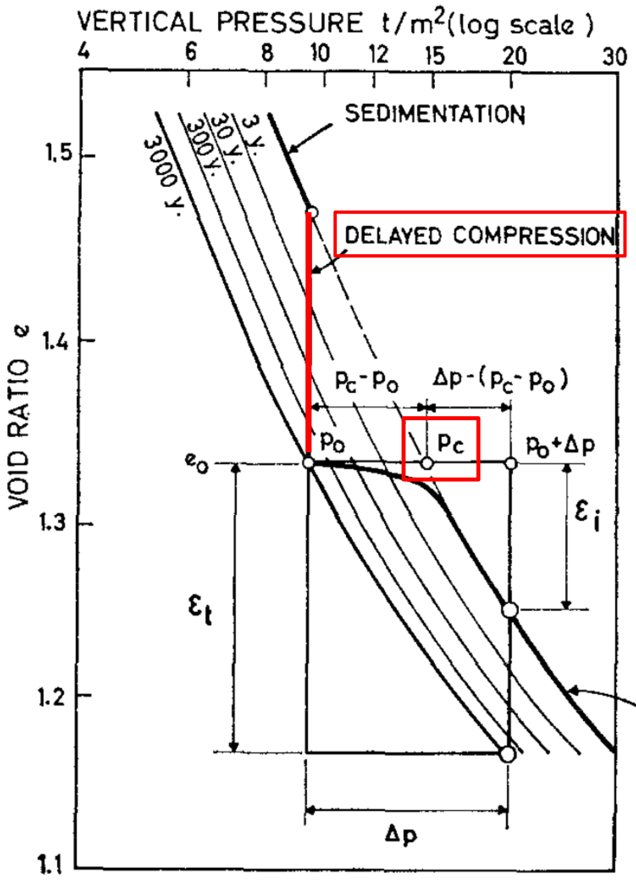
Creep rate increases with:
- **Plasticity**
- **Activity**
- **Water content**

Sketch after Mitchel & Soga 2005

Oedometric testing

Apparent preconsolidation pressure

- Compression at constant effective stress: for instance due to the gain of resistance against compression
- Apparent preconsolidation: New preconsolidation pressure obtained after a consolidation at constant pressure



ε_i = instant compression $= \frac{C_c}{1+e_o} \log \frac{p_c + [\Delta p - (p_c - p_o)]}{p_c}$

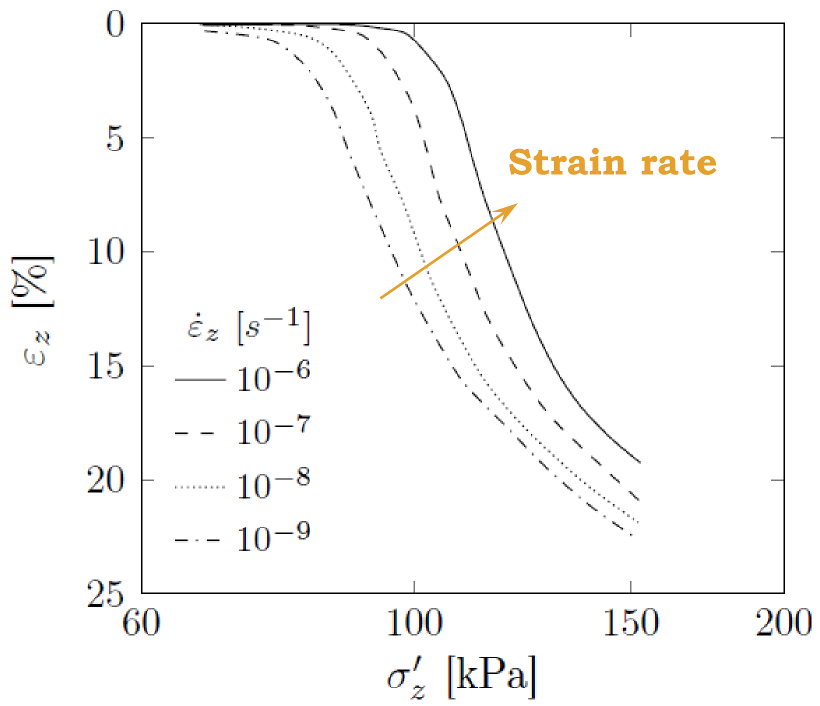
ε_t = total (instant + delayed) compression after 3000 y. $= \frac{C_c}{1+e_o} \log \frac{p_o + \Delta p}{p_o}$

Bjerrum (1967)

→ The apparent preconsolidation is not due to the overburden experienced by the material anymore, but reflects the gain of stiffness/resistance to compression from the consolidation at constant pressure

Oedometric testing

The same material response can be obtained with a constant rate of strain oedometric loading (CRS).
The increase of the applied strain rate leads to the development of an apparent preconsolidation stress.



Leroueil and Kabbaj (1985)

- The increase of the observed preconsolidation stress is almost linear with the logarithm of the applied strain rate

Rate and Time Dependency

- Soils can display a **highly viscous behaviour**
- The main macroscopical displays of such behaviour are:
  a. **strain rate effect** (rate dependency)
  b. **creep** (time dependency of strains)
  c. **relaxation** (time dependency of stresses)
- Various macroscopical experimental observations all express **similar fundamental microscopic processes**

Long term resistance ⇨ very small strain rate ⇨ elastoplasticity

⇨ **Time-dependent behaviour (Argotropy)**

Constitutive modelling
Visco-elasto-plasticity

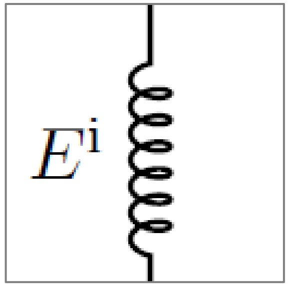
Constitutive modelling

The viscous deformations can be **ELASTIC** or **PLASTIC**

1) Viscoelastic models

2) Elastic visco-plastic models

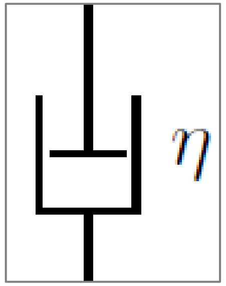
Elastic deformations are modelled with **springs**



$$\varepsilon^{ie} = \frac{\sigma}{E_i}$$

E_i: elastic stiffness

Viscous deformations are modelled with **dampers**



$$d\varepsilon^{v} = \frac{\sigma}{\eta} dt$$

η: viscosity

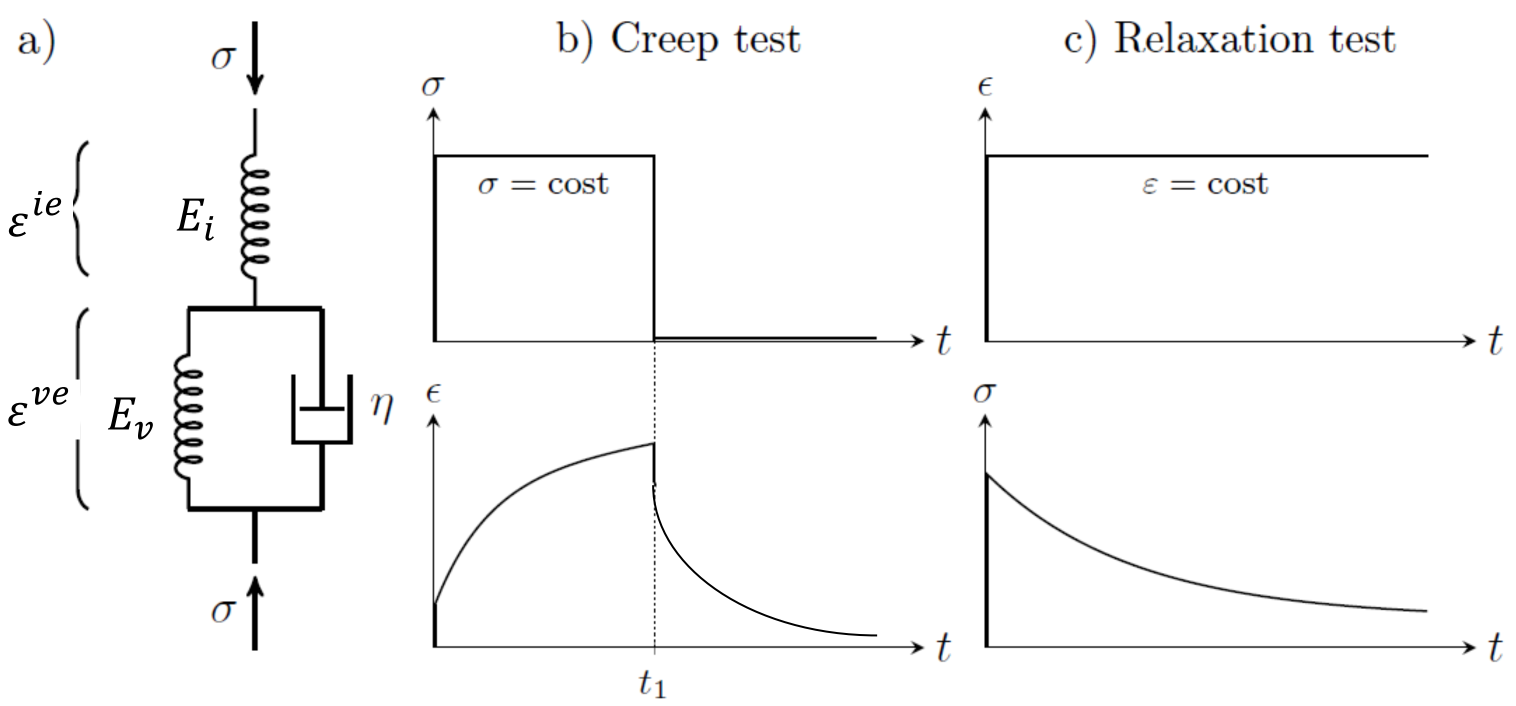
Viscous model

Standard model

Two elements: the **spring**, and the **Kelvin-Voigt unit** connected in series

Instantaneous elastic deformation (reversible)

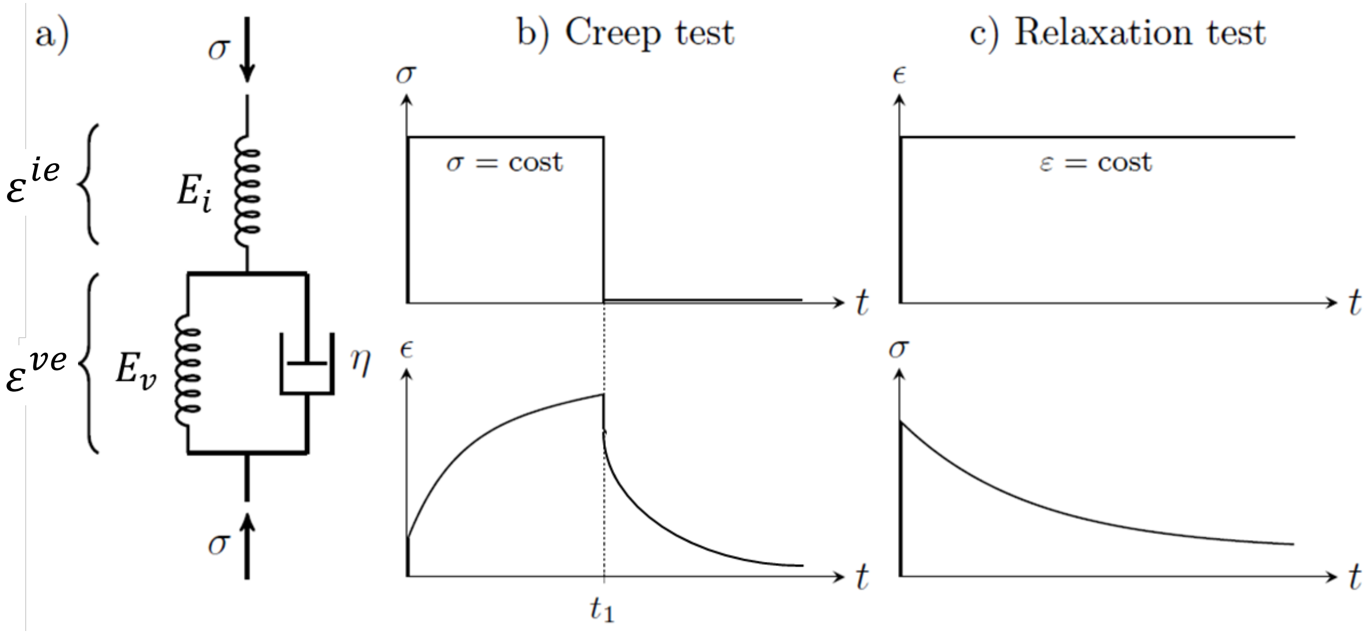
Spring and dumper connected in parallel
→ **Viscous deformation** (reversible)



Viscous model

Standard model

Two elements: the **spring**, and the **Kelvin-Voigt unit** connected in series



Same stress σ

$$\sigma = E_i \quad \varepsilon^{ie} = \sigma = E_v \quad \varepsilon^{ve} + \eta \dot{\varepsilon}^{ve}$$

Cumulative strain ε

$$\varepsilon = \varepsilon^{ie} + \varepsilon^{ve} = \frac{\sigma}{E_i} + \frac{\sigma}{E_v}(1 - \exp(-\frac{t}{t_r}))$$

with $$t_r = \frac{\eta}{E_v}$$

Elastic visco-plastic model

Maxwell model

Spring and dumper in series

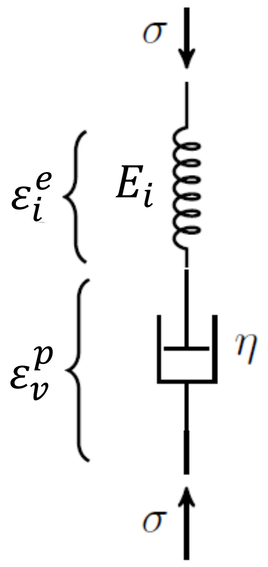
Plastic deformation induced by the dumper are irreversible (ε_p^v)

total strain

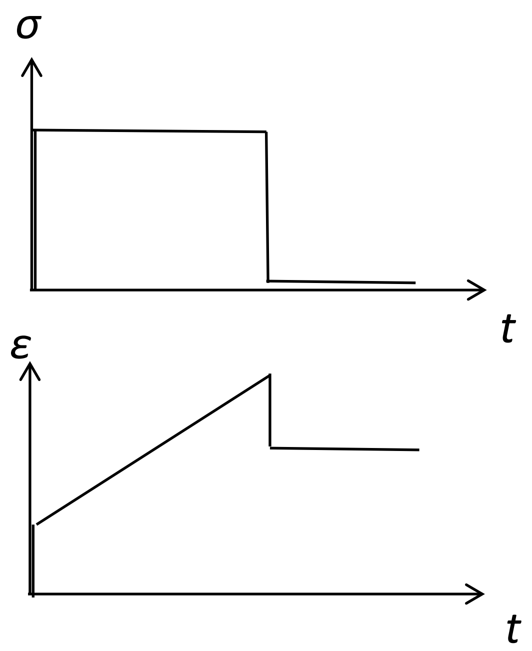
$$\varepsilon = \varepsilon^{ie} + \varepsilon^{vp}$$

elastic instantaneous strain — plastic viscous strain

$$\varepsilon = \frac{\sigma}{E_i} + \frac{\sigma}{\eta} t$$



Creep test



Elastic visco-plastic model

Bingham model

- A frictional element is introduced in parallel with a damper
- A yield stress σ_{pc} has to be exceeded in order to have plastic viscous deformations ε_p^v

total strain ↑

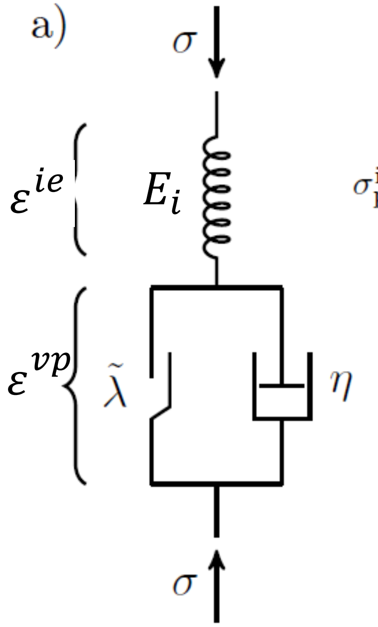
$$\varepsilon = \varepsilon^{ie} + \varepsilon^{vp}$$

↓ elastic instantaneous strain

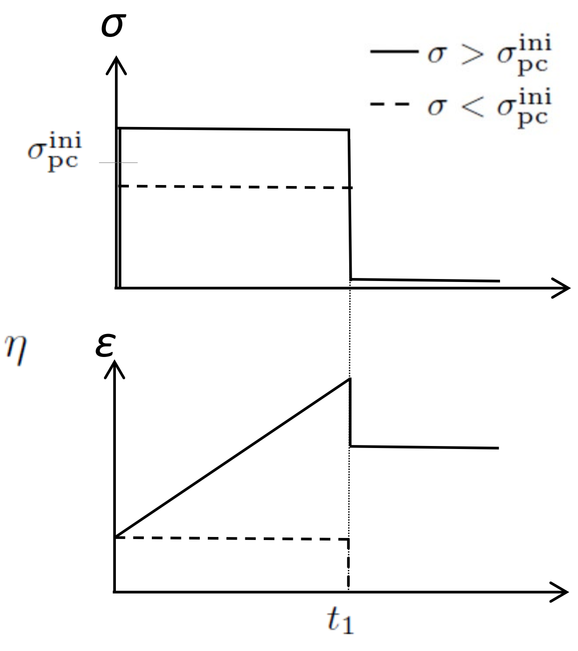
↓ plastic viscous strain

If $\sigma < \sigma_{pc}$ → $\varepsilon = \varepsilon^{ie} = \dfrac{\sigma}{E_i}$

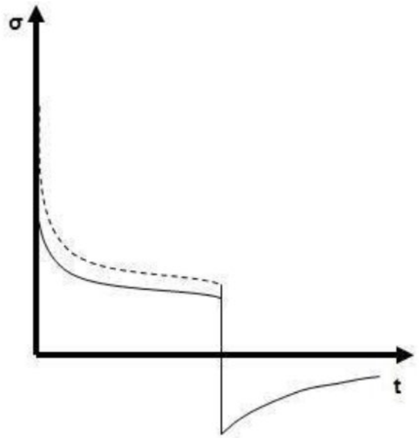
if $\sigma > \sigma_{pc}$ → $\varepsilon = \dfrac{\sigma}{E_i} + \dfrac{\sigma - \sigma_{pc}}{\eta} t$



Creep test



Relaxation test



Advanced visco-plastic model

→ For the modelling, we are no more interested in the evaluation of strains but of **strain rates**

Total strain rate

$$\dot{\varepsilon} = \underline{\underline{\dot{\varepsilon}}}^{e} + \underline{\underline{\dot{\varepsilon}}}^{vp}$$

Viscoplastic strain rate

$$\dot{\varepsilon}^{e} = \boldsymbol{C} : \dot{\boldsymbol{\sigma}}'$$

Elastic strain rate

$$\dot{\varepsilon}^{vp} = \lambda \frac{\partial g}{\partial \boldsymbol{\sigma}'}$$

- Perzyna Framework
- Consistency framework
- Secondary consolidation

→ Viscoplastic model is built with the same elements as elasto-plastic models (i.e. Modified Cam Clay model)

- Elastic behaviour
- Yield function
- Plastic potential and flow rule
- Hardening rule

Advanced visco-plastic model

Conventional Perzyna approach (Perzyna, 1966)

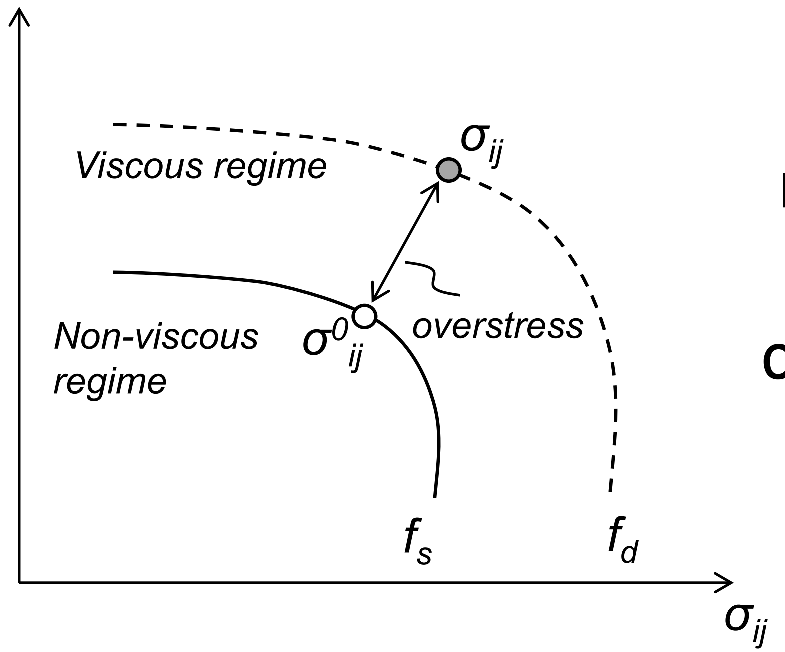
- Use of a **rate-independent** yield function that can become **larger than zero**

→ New yield function for viscoplasticity: **Overstress function**

- Within the usual yield limit f_s, the response is **purely elastic**
- Beyond the usual yield limit f_s, the response is **viscoplastic**

Problem n°1:
Consistency condition is violated

Problem n°2:
Experimental evidence of strain rate dependency



Rate-independent yield function $f(\sigma', \varepsilon_v^{vp})$

Overstress function $$\Phi(f) = \left(\frac{f}{f_0}\right)^N$$

Flow rule $$\dot{\varepsilon}^{vp} = \frac{\langle \Phi(f) \rangle}{\eta} \frac{\partial g}{\partial \sigma'}$$

Advanced visco-plastic model

Consistency approach (Wang, 1997)

- Introduction of variables making the yield function **rate-dependent**
- The rate-dependent yield function governs the irreversible viscoplastic strain

  → extension of the classical elasto-plastic approach
- Viscoplastic strain rate is **implicitely** determined via the **rate-dependent consistency condition**

Rate-independent yield function

$$\boldsymbol{f}\left(\boldsymbol{\sigma}', \dot{\boldsymbol{\varepsilon}}_v^{vp}, \boldsymbol{\varepsilon}_v^{vp}\right)$$

Flow rule

$$\dot{\varepsilon}^{vp} = \dot{\lambda}\frac{\partial g}{\partial \sigma'}$$

Consistency equation

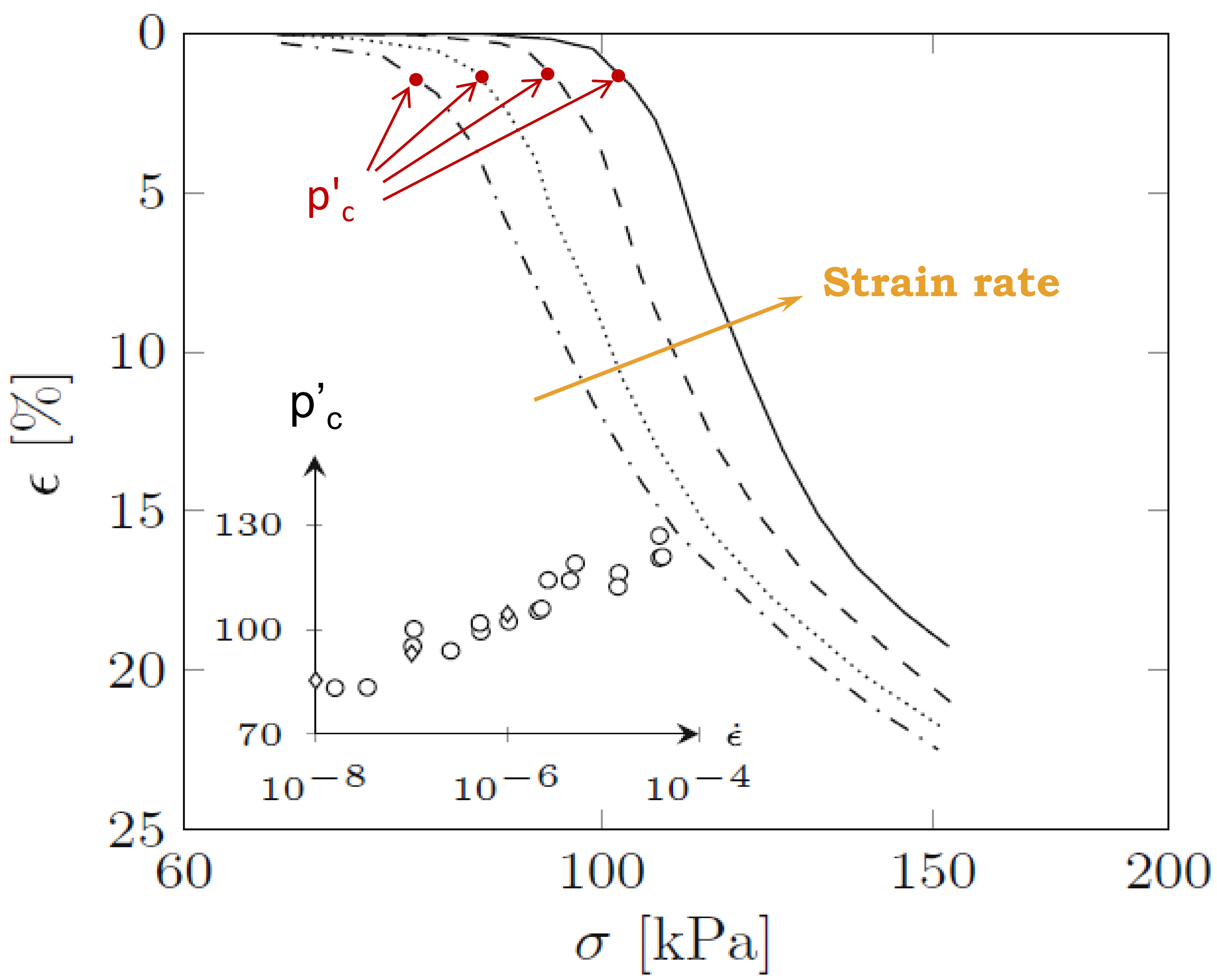
$$\dot{f}^{rd} = \frac{\partial f^{rd}}{\partial \boldsymbol{\sigma}'} : \dot{\boldsymbol{\sigma}}' + \frac{\partial f^{rd}}{\partial \varepsilon_v^{vp}} \frac{\partial \varepsilon_v^{vp}}{\partial \lambda} . \dot{\lambda} + \frac{\partial f^{rd}}{\partial \dot{\varepsilon}_v^{vp}} \frac{\partial \dot{\varepsilon}_v^{vp}}{\partial \dot{\lambda}} . \ddot{\lambda} \leq 0$$

Advanced visco-plastic model

1. **Observation** → Different preconsolidation pressure obtained for the same material at different strain rates (CRS tests)

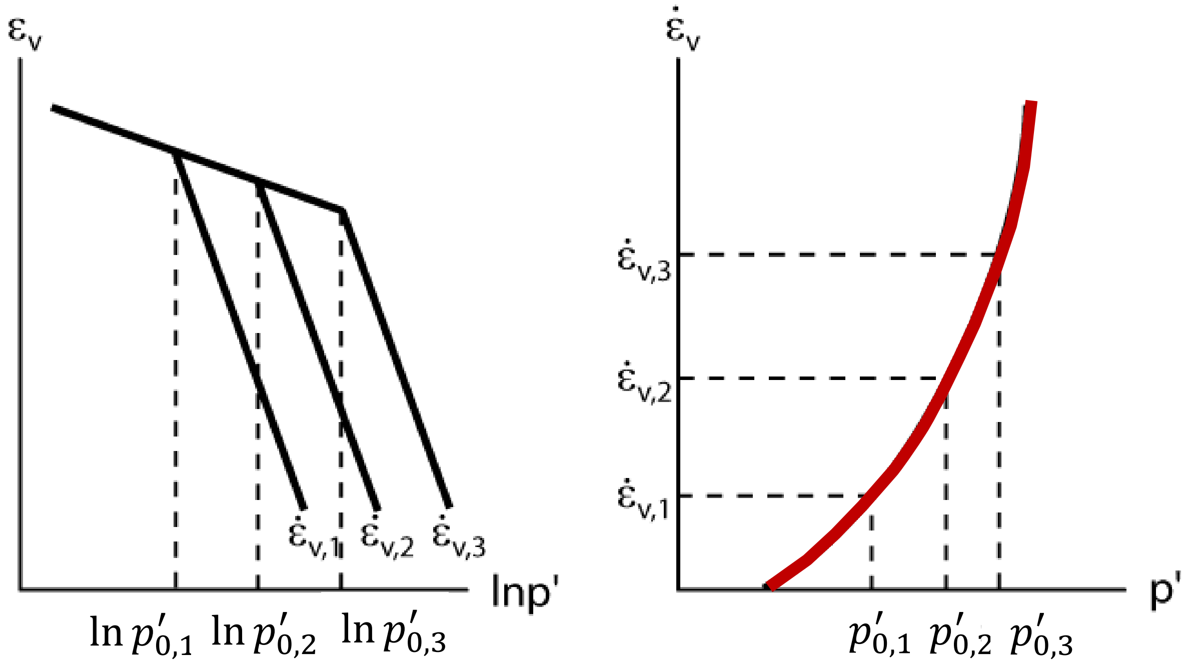
2. **Hypothesis** → Preconsolidation pressure p_c' depends on the applied strain rate

3. **Mathematical formulation:** Preconsolidation expressed in terms of the strain rate



Advanced visco-plastic model

- The unique **vertical effective stress – vertical viscoplastic strain – vertical viscoplastic strain rate concept** is expressed through the evolution law of the apparent preconsolidation pressure with viscoplastic strain rate



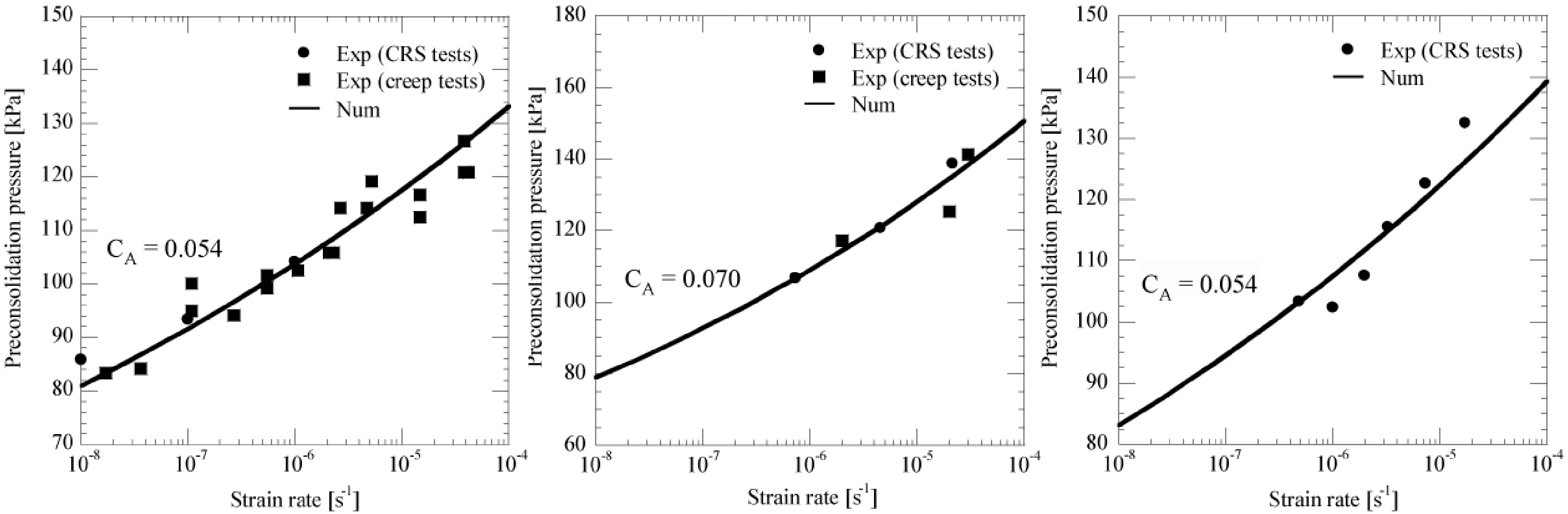
$$p'_{0,\dot{\varepsilon}_v^{vp}} = p'_{0,\dot{\varepsilon}_{v0}^{vp}} \left(\frac{\dot{\varepsilon}_v^{vp}}{\dot{\varepsilon}_{v,ref}^{vp}} \right)^{C_A}$$

Leroueil et al. (1985)

Advanced visco-plastic model

Performances of the evolution law:

$$p'_{0,\dot{\varepsilon}_v^{vp}} = p'_{0,\dot{\varepsilon}_{v0}^{vp}} \left(\frac{\dot{\varepsilon}_v^{vp}}{\dot{\varepsilon}_{v,ref}^{vp}} \right)^{C_A}$$



Batiscan clay (Leroueil et al., 1985) Bäckebol clay (Leroueil et al., 1985) St Césaire clay (Leroueil et al., 1985)

Advanced visco-plastic model

Apparent preconsolidation pressure

Hardening variable

$$p'_{0,\dot{\varepsilon}_{v0}^{vp}} = p'_{0,\dot{\varepsilon}_{v0}^{vp},0}\exp(\beta \varepsilon_v^{vp})$$

Evolution of the yield surface with the plastic strain

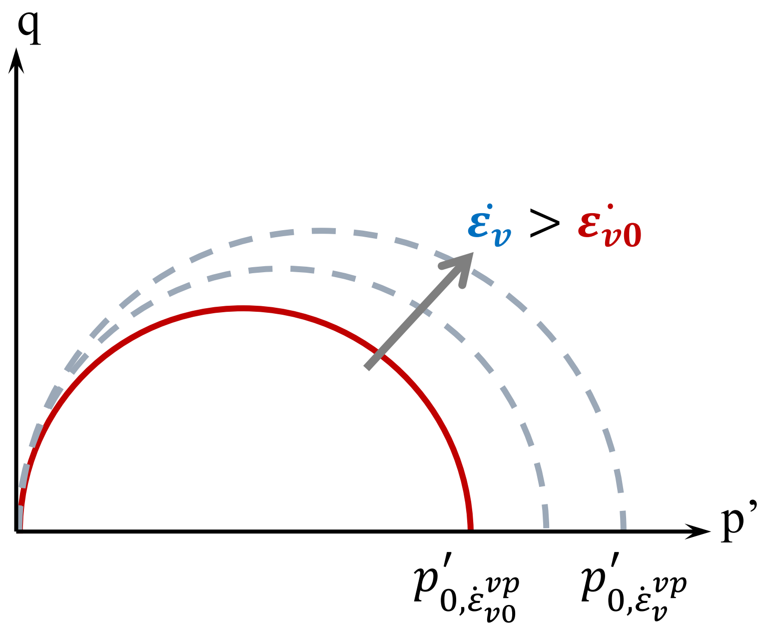
Time effect

$$p'_{0,\dot{\varepsilon}_v^{vp}} = p'_{0,\dot{\varepsilon}_{v0}^{vp}}\left(\frac{\varepsilon_v^{vp}}{\varepsilon_{v,ref}^{vp}}\right)^{C_A}$$

Dependency of the $p'_{c,\dot{\varepsilon}_v^{vp}}$ with the strain rate

- This rate-dependent preconsolidation pressure is then introduced in the yield function formulation

Cam Clay type model



$$f\left(\sigma', \dot{\varepsilon}_v^{vp}, \varepsilon_v^{vp}\right)$$

Advanced visco-plastic model

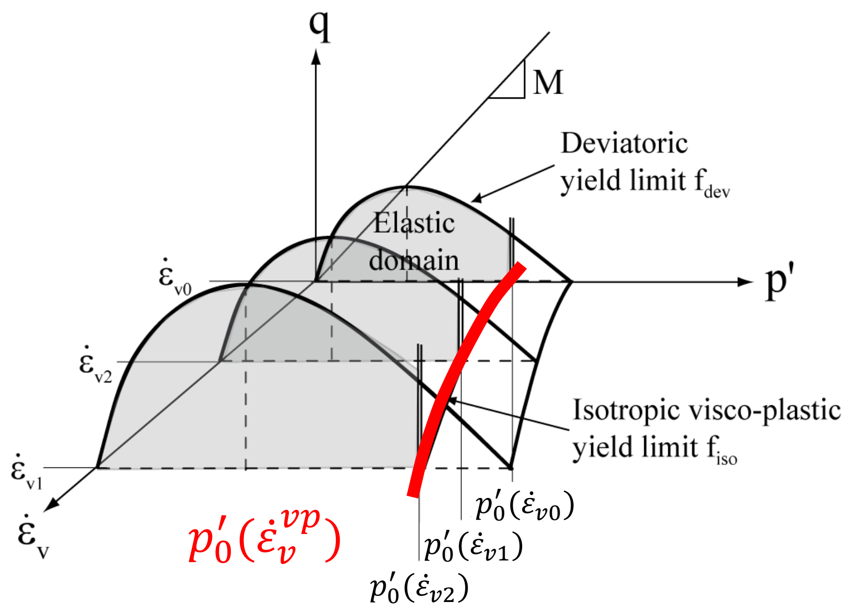
ACMEG – VP model (developed at the LMS-EPFL)

- Elastic behaviour $\dot{\varepsilon} = \dot{\varepsilon}_v^e + \dot{\varepsilon}_d^e$ $\quad \dot{\varepsilon}_v^e = \frac{\dot{p}'}{K}$ $\quad \dot{\varepsilon}_d^e = \frac{\dot{q}}{3G}$
- Yield function $f_{iso} = p' - p'_0 = 0$ $\quad f_{dev} = q^2 - M^2\left[p'^2\left(1 + b^2 ln^2 \frac{p'd}{p'_0}\right)\right]$
- Hardening rule $p'_0 = p'_{0,ref} \times \left(\frac{\dot{\varepsilon}_v^{vp}}{\dot{\varepsilon}_{v,ref}^{vp}}\right)^{C_A} \times \exp(\beta \varepsilon_v^{vp})$
- Potential function $g_{iso} = p' - p'_0$

$$g_{dev} = q - \frac{\alpha}{\alpha - 1} Mp\left(1 - \frac{1}{\alpha}\left(\frac{pd}{p'_0}\right)^{\alpha-1}\right)$$

- Consistency condition $\dot{\lambda}^{vp}\dot{f} = 0$

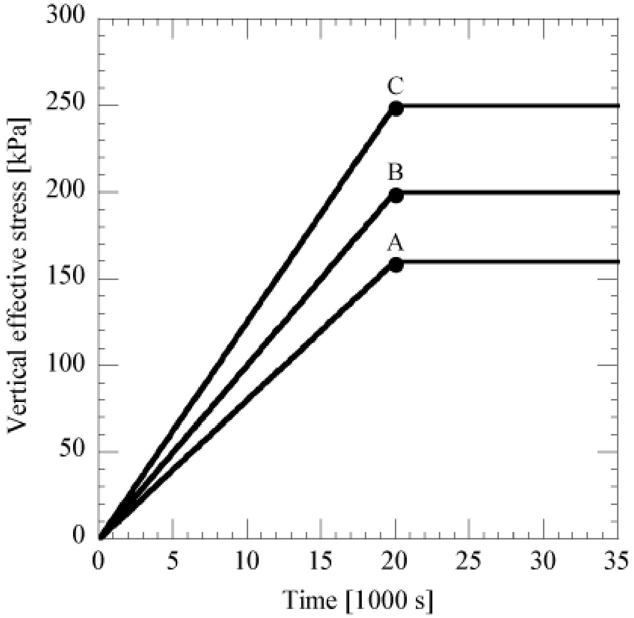
$$\dot{f} = \frac{\partial f}{\partial \sigma'} : \dot{\sigma}' + \underbrace{\frac{\partial f}{\partial \varepsilon_v^{vp}} \frac{\partial \varepsilon_v^{vp}}{\partial \lambda^{vp}}}_{-H} .\dot{\lambda}^{vp} + \underbrace{\frac{\partial f}{\partial \dot{\varepsilon}_v^{vp}} \frac{\partial \dot{\varepsilon}_v^{vp}}{\partial \dot{\lambda}^{vp}}}_{-S} .\ddot{\lambda}^{vp} = 0$$

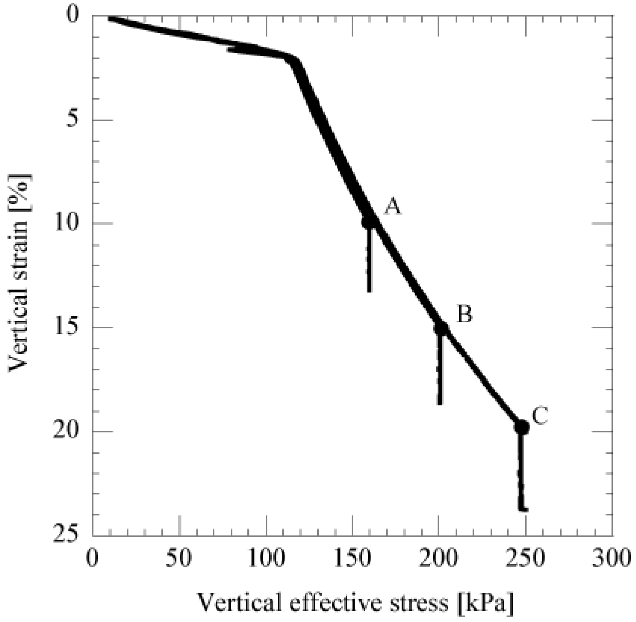


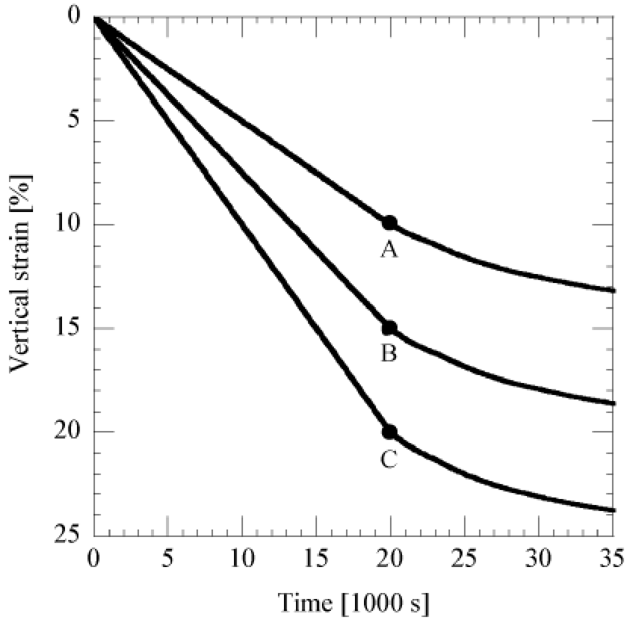
Advanced visco-plastic model

Numerical example of a creep test

- Creep loading





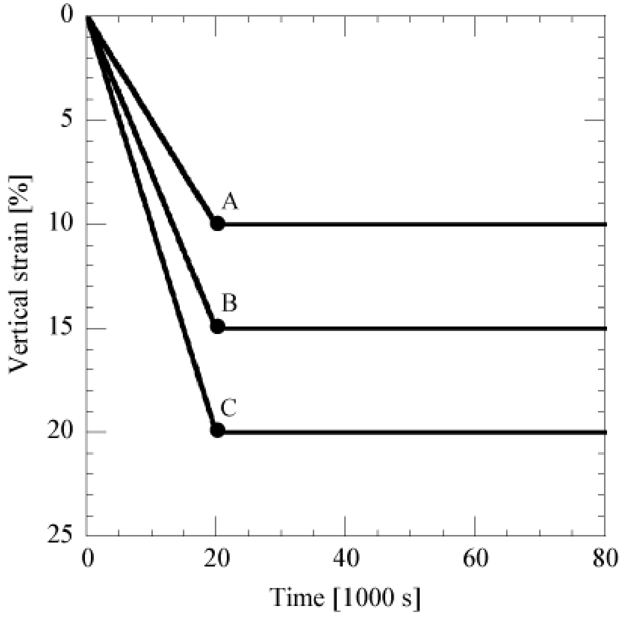


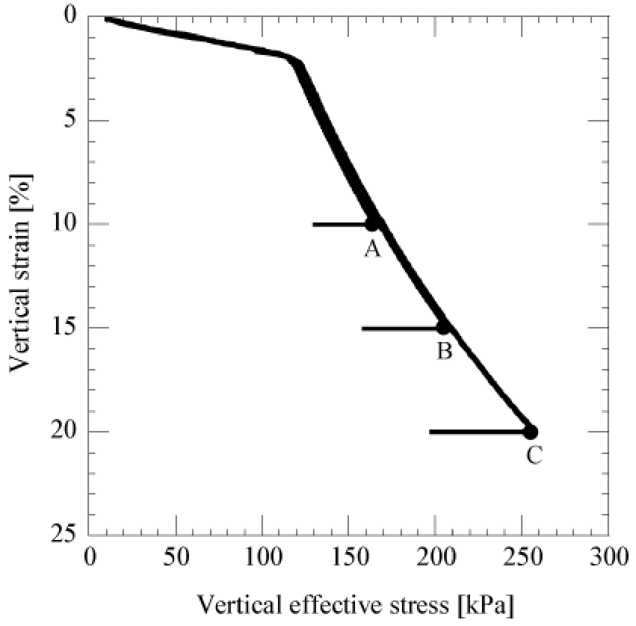
- Qualitatively good reproduction of creep behaviour
- Modelling of creep behaviour possible with the consistency approach

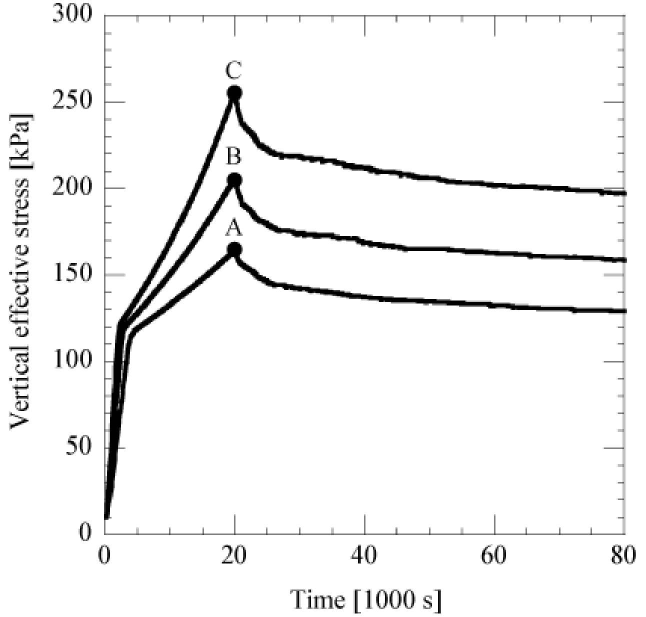
Advanced visco-plastic model

Numerical example of a **relaxation test**

- Relaxation loading





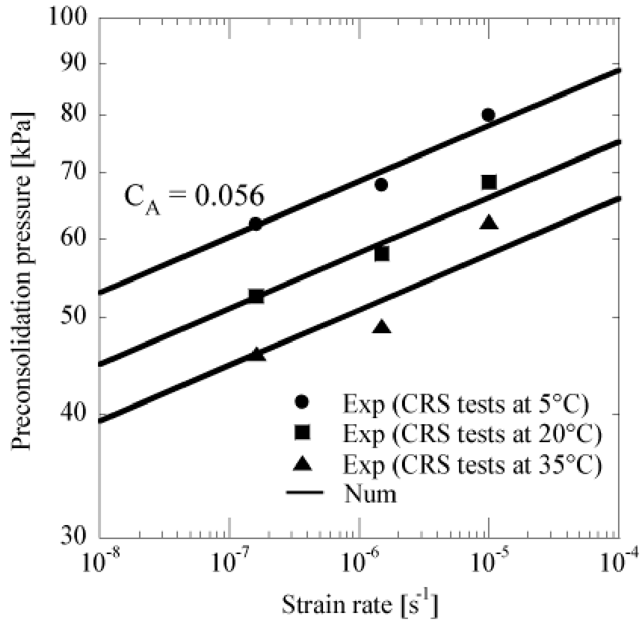


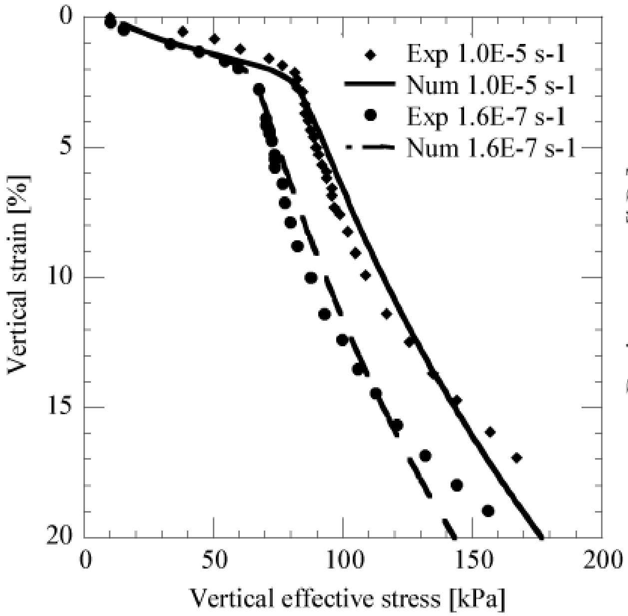
- Qualitatively good reproduction of relaxation behaviour
- Modelling of relaxation behaviour possible with the consistency approach

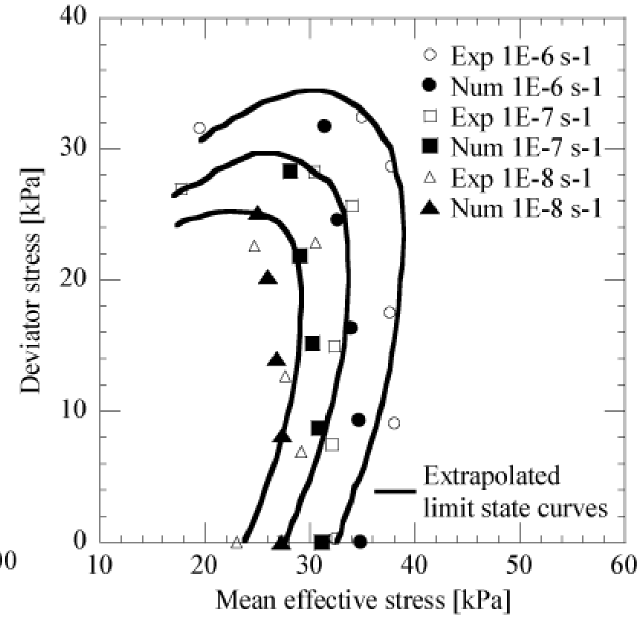
Advanced visco-plastic model

Validation on a CRS oedometric test

- Constant rate of strain oedometric loading
- Calibration of elasto-plastic model parameters on the CRS consolidation curve at 1.6E-7 s^{-1}
- Determination of the materiel parameter C_A by curve fitting on available experimental points







Elastic	K	[MPa]	16
	G	[MPa]	9.6
	n	[-]	0.5
Plastic	β	[-]	4.6
	p'_c	[kPa]	32
	φ	[%]	25
Viscous	C_A	[-]	0.056

Berthierville clay
(Boudali et al., 1994)

Conclusion

Conclusion

- Hydro-mechanical and viscous response are two distinct phenomena
- Inappropriate analysis of these phenomena leads to misleading evaluation of the material response
- Viscous response is more important in clays
- Viscous behaviour can be studied both in oedometric and triaxial tests

Conclusion

- Viscous deformation can be reversible (visco-elastic behaviour) or non-reversible (visco-plastic behaviour)
- Strength is affected by strain rate
- Preconsolidation pressure increases with the applied strain rate
- Apparent preconsolidation pressure is developed during constant load application